

## RUNNING MASS OF THE $B$ -QUARK IN QCD AND SUSY QCD

A.V. BEDNYAKOV

*BLTP, Joint Institute for Nuclear Research, Dubna, Russia*  
*bednya@theor.jinr.ru*

The running mass of the  $b$ -quark defined in  $\overline{\text{DR}}$ -scheme is one of the important parameters of SUSY QCD. To find its value, it should be related to some known experimental input. In this paper, the  $b$ -quark running mass defined in nonsupersymmetric QCD is chosen for determination of the corresponding parameter in SUSY QCD. The relation between these two quantities is found by considering five-flavor QCD as an effective theory obtained from its supersymmetric extension. A numerical analysis of the calculated two-loop relation and its impact on the MSSM spectrum is discussed. Since for nonsupersymmetric models  $\overline{\text{MS}}$ -scheme is more natural than  $\overline{\text{DR}}$ , we also propose a new procedure that allows one to calculate relations between  $\overline{\text{MS}}$ - and  $\overline{\text{DR}}$ -parameters. Unphysical  $\varepsilon$ -scalars that give rise to the difference between the above-mentioned schemes are assumed to be heavy and decoupled in the same way as physical degrees of freedom. By means of this method it is possible to “catch two rabbits”, i.e., decouple heavy particles and turn from  $\overline{\text{DR}}$  to  $\overline{\text{MS}}$ , at the same time. An explicit two-loop example of  $\overline{\text{DR}} \rightarrow \overline{\text{MS}}$  transition is given in the context of QCD. The advantages and disadvantages of the method are briefly discussed.

*Keywords:* QCD; MSSM;  $b$ -quark

PACS numbers: 12.38.-t, 12.38.Bx, 12.60.Jv, 14.65.Fy

### 1. Introduction

It is commonly believed that the Standard Model (SM) is not the ultimate theory of particle physics. Among other deficiencies of the SM there is so-called fine tuning problem which arises from quadratic dependence of the Higgs mass on the new physics scale.

A popular extension of the SM that cures this problem is the Minimal Supersymmetric Standard Model (MSSM). The construction of the CERN Large Hadron Collider (LHC) has led to many increasingly precise calculations of sparticle production and decay processes.

An important ingredient of the model is the SUSY QCD sector. In most of the processes with color particles radiative corrections from the strong interactions give a dominant contribution. Loop corrections to tree-level processes in SUSY QCD are usually expressed in terms of running parameters defined in so-called  $\overline{\text{DR}}$ -scheme. It is an analog of  $\overline{\text{MS}}$  renormalization scheme based on Dimensional Reduction (DRED)<sup>1</sup>.

The running mass  $m_b^{\overline{\text{DR}}}$  of the  $b$ -quark is one of the important parameters of SUSY QCD. The value of  $m_b^{\overline{\text{DR}}}$  at a scale  $\mu$  should be obtained from some experimental input. However, for the  $b$ -quark it is very hard to find or even define such an input. The pole mass  $M_b$  being very well defined in a finite order of perturbative QCD<sup>2,3</sup> suffers from renormalon ambiguity<sup>4,5</sup> that gives rise to  $\Lambda_{\text{QCD}}/m_b \sim 10\%$  uncertainty in its determination. There is another issue in using the pole mass for determination of  $m_b^{\overline{\text{DR}}}$ . The relation between these two quantities exhibits a logarithmic dependence on all mass scales of SUSY QCD. This is a typical, non-decoupling, property of minimal renormalization schemes. In our problem we have very different scales, i.e.,  $m_b \ll M_{\text{SUSY}}$ . Consequently, one cannot make all the logarithms small by some choice of the renormalization scale  $\mu$ , thus leading to inaccurate perturbative prediction for  $m_b^{\overline{\text{DR}}}$ .

A convenient quantity to use for extraction of  $m_b^{\overline{\text{DR}}}$  is the  $b$ -quark running mass  $m_b^{\overline{\text{MS}}}$  defined in the five-flavor QCD renormalized in  $\overline{\text{MS}}$ -scheme<sup>6</sup>. The value of the running mass at the scale which is equal to itself is known from PDG<sup>7</sup>,  $m_b^{\overline{\text{MS}}}(m_b^{\overline{\text{MS}}}) = 4.20 \pm 0.07$  GeV.

In this paper, we calculate an explicit two-loop relation between  $m_b^{\overline{\text{MS}}}$  and  $m_b^{\overline{\text{DR}}}$  by the so-called matching procedure (see, e.g., Refs. 8 and 9).

In Section 2, our theoretical framework is described. Renormalizable QCD is considered as an effective theory that can be obtained from a more fundamental<sup>a</sup> one by decoupling of heavy particles. Decoupling of heavy degrees of freedom manifests itself in relations between parameters of fundamental and effective theories. Intuitively, in the energy region far below the corresponding threshold contribution of virtual heavy loops to the light particle effective action can be approximated by local *renormalizable* operators that respect gauge invariance and, consequently, can be absorbed into redefinition of Lagrangian parameters and fields.

In Section 3, we discuss how relations between  $\overline{\text{MS}}$ -parameters and their counterparts in  $\overline{\text{DR}}$ -scheme can be obtained on the same footing. The distinction between  $\overline{\text{MS}}$  and  $\overline{\text{DR}}$  essentially comes from the presence of so-called  $\varepsilon$ -scalars. Since the scalars are unphysical, we may assume that they are heavy and decouple them in almost the same way as one decouples physical degrees of freedom. As an example of the formalism, in Section 4, we consider two-loop matching of  $\overline{\text{DR}}$  QCD with  $\overline{\text{MS}}$  QCD. We show how known relations between  $\overline{\text{DR}}$  and  $\overline{\text{MS}}$  QCD parameters (see, e.g., Ref. 10) are reproduced.

In Section 5, we use the described technique to calculate a two-loop relation between  $\overline{\text{MS}}$  and  $\overline{\text{DR}}$  running masses of the  $b$ -quark in QCD and SUSY QCD part of the MSSM. In our approach, we decouple all heavy particles simultaneously (“common scale approach” of Ref. 11). For the TeV-scale SUSY it seems phenomenologically acceptable since the electroweak scale is usually used for matching. Simultaneous decoupling results in a lengthy final expression. As a consequence, only

<sup>a</sup> In what follows we use adjectives “full”, “fundamental” and “high-energy” as synonyms to distinguish a more fundamental theory from the effective one

numerical impact of the result is presented.

In our calculations we made use of FeynArts<sup>12</sup> to generate needed Feynman amplitudes. Since  $\overline{\text{DR}} \rightarrow \overline{\text{MS}}$  transition requires explicit treatment of  $\varepsilon$ -scalars, the interaction Lagrangian for the unphysical fields was implemented. Some details of the implementation can be found in a series of appendices.

## 2. Decoupling of heavy particles and Large Mass Expansion

In QCD and its supersymmetric extension it is convenient to use mass-independent (minimal subtractions or MS) renormalization schemes. In these schemes beta-functions and anomalous dimensions have a very simple structure. However, physical quantities expressed in terms of the running parameters exhibit a nonanalytic logarithmic dependence on all mass scales of the theory.

If there is a big hierarchy between mass scales, it is not satisfactory, since due to this nonanalytic mass dependence, a contribution of heavy degrees of freedom to low-energy observables is not suppressed by the inverse power of the corresponding heavy mass scale. It is said that in MS-schemes in contrast to momentum subtraction schemes (MOM), the Appelquist-Carrazone decoupling theorem<sup>13</sup> does not hold.

A proper way to overcome the above-mentioned difficulties of MS-schemes is to use effective (low-energy) theories to describe physics at relevant energy scales  $E$ . If at given energies heavy particles with mass  $M > E$  can only appear in virtual states one may use an effective field theory with the corresponding heavy fields omitted.

In a general case low-energy theories are not renormalizable. Moreover, to reproduce physics close to the threshold  $E \lesssim M$  correctly, they should contain infinitely many nonrenormalizable interactions parametrized by dimensionful couplings. However, given a more fundamental theory one can relate all the couplings in the effective low-energy Lagrangian to fundamental parameters of the high-energy theory. Roughly speaking, one should calculate observables (or, more strictly, Green functions) in both theories and tune the parameters and field normalization in the effective Lagrangian in such a way that both results coincide in the region below the threshold.

This procedure is called matching. As a result of matching one expresses effective theory parameters as functions of fundamental ones. In practice, one cannot deal with an infinite number of interactions. So one usually performs *asymptotic* expansion of Green functions defined in high-energy theory in  $E/M$  and demands that effective theory should correctly reproduce a finite order of the expansion considered.

In some cases effective theories are used for energies far below the threshold so a contribution from nonrenormalizable operators is usually suppressed by powers of  $E/M$ . One may even consider *renormalizable* effective theory. In this case, the structure of low-energy Lagrangian differs from the one of the fundamental theory only by omission of all the heavy fields and their interactions.

One usually says that decoupling of heavy particles is manifest if one can directly

use the parameters defined in a fundamental theory to calculate quantities within the effective theory (and vice versa). In a momentum subtraction scheme decoupling is obvious, since all the parameters are defined through Green functions evaluated at certain external momenta (less than  $M$ ). Since we demand that Green functions in both theories coincide, all the parameters defined in such a way also coincide.

On the contrary,  $\overline{\text{MS}}$ -parameters are not related to Green functions evaluated at some finite momenta. Thus, one does not expect that  $\overline{\text{MS}}$ -parameters have the same value in both the theories. In this sense decoupling in  $\overline{\text{MS}}$ -schemes does not hold. One should manually tune the parameters defined in such a scheme.

How does the approach based on effective theories help one to avoid the appearance of large logarithms for  $E \ll M$  in a calculation? The trick is to separate  $\log M/E$  into  $\log M/\mu$  and  $\log E/\mu$ , where  $\mu$  is an arbitrary separation scale which in  $\overline{\text{MS}}$ -scheme is naturally equal to the 'tHooft unit mass. Then one may absorb  $\log M/\mu$  into low-energy parameters of the effective theory by a matching procedure. A typical relation between the parameters is

$$\underline{A}(\mu) = A(\mu) \times \zeta_A(A(\mu), B(\mu), M, \mu), \quad (1)$$

where  $\underline{A}$  is a dimensionless parameter of the effective theory<sup>b</sup>,  $A$  denotes its counterpart in the high-energy theory, and  $B$  are other dimensionless fundamental theory parameters. The function  $\zeta_A$  is called decoupling constant for  $A$ . Generalization of (1) to dimensionful parameters is straightforward. However, one should keep in mind that the crucial property of  $\zeta_A$  is its independence of small energy scales. One calculates  $\zeta_A$  in perturbation theory and at the tree level  $\zeta_A = 1$ . Given  $\mu \sim M$  one avoids large logarithms in (1). If one knows fundamental parameters at the scale  $\mu \sim M$ , one can find the values of the effective-theory parameters at the same scale. Clearly, direct application of the parameters  $\underline{A}(\mu \sim M)$  in low-energy theory again introduces large logarithms  $\log E/\mu \sim \log E/M$ . However, in the effective theory it is possible to sum these logarithms by the renormalization group method, i.e., going from  $\underline{A}(M)$  to  $\underline{A}(E)$  (see Fig. 1).

Actually, one usually reverses the reasoning. Typically, fundamental theory parameters are unknown (especially, the value of  $M$ ), but one knows the value of  $\underline{A}(E)$  normalized at some low-energy scale  $E$ . Direct application of (1) again introduces large logarithms in the right-hand side,  $\log M/\mu \sim \log M/E$ . As in the previous case, one should use renormalization group equations defined in the effective theory to evaluate  $\underline{A}(\mu)$  at  $\mu \sim M$ . Consequently, relation (1) can be interpreted as a constraint on the fundamental theory parameters. Usually, largest variations of the right-hand side of (1) come from variations of  $A$  parameter, so one says that the value of  $A(\mu)$  is extracted from  $\underline{A}(\mu)$ . It is this type of relations we are interested in. Let us describe the procedure that one can use to calculate decoupling (matching) corrections.

<sup>b</sup>In what follows we underline effective theory parameters and fields

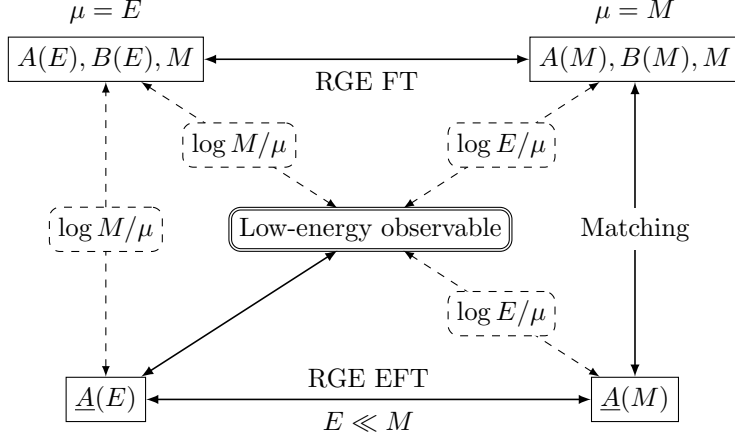


Fig. 1. The diagram shows various relations between full theory (FT) running parameters ( $A, B, M$ ), the parameters  $\underline{A}$  of the effective theory (EFT), and low-energy observables. Renormalization group equations (RGE) together with matching at the scale  $M$  allows one to avoid the appearance of large logarithms (see dashed boxes) in calculation. Solid lines correspond to relations that do not introduce large logarithms explicitly.

Consider the Lagrangian of a full theory,  $\mathcal{L}_{full}$ . For the moment, we do not specify it explicitly. The crucial property of the theory is that it describes not only gluons and quarks (light fields denoted by  $\phi$ ), but also heavy fields  $\Phi$  with typical masses  $M$ :

$$\mathcal{L}_{full} = \mathcal{L}_{QCD}(\phi, g_s, m, \xi) + \Delta\mathcal{L}(\phi, \Phi, g_s, m, M, \dots). \quad (2)$$

Here  $\phi \equiv \{G_\mu, q_{L,R}, c\}$  are gluon, quark, and ghost fields, respectively. The strong gauge coupling is denoted by  $g_s$ ,  $m$  corresponds to quark masses and  $\xi$  is a gauge-fixing parameter. It should be noted that  $\Delta\mathcal{L}$  represents the Lagrangian for heavy fields and contains kinetic terms for  $\Phi$  together with various interactions<sup>c</sup>. The QCD Lagrangian  $\mathcal{L}_{QCD}$  has the usual form:

$$\begin{aligned} \mathcal{L}_{QCD} = & -\frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu} + \bar{q} (i\hat{D} - m) q, \\ & - \frac{1}{2\xi} (\partial_\mu G_\mu^a)^2 - \bar{c}^a \partial^\mu (\partial_\mu \delta^{ab} + g_s f^{abc} G_\mu^c) c^b, \end{aligned} \quad (3)$$

where

$$\begin{aligned} F_{\mu\nu}^a &= \partial_\mu G_\nu^a - \partial_\nu G_\mu^a - g_s f^{abc} G_\mu^b G_\nu^c, \\ D_\mu &= \partial_\mu + ig_s G_\mu, \quad G_\mu = T^a G_\mu^a, \\ [T^a, T^b] &= if^{abc} T^c, \quad \text{Tr } T^a T^b = T_F \delta^{ab}, \quad T_F = 1/2 \end{aligned}$$

and for simplicity we omit summation over quark flavours. For energies below the

<sup>c</sup>In this paper, we only consider strong interactions between all fields of the fundamental theory

threshold  $E < M$  one is interested in Green functions with light external fields

$$\langle T\phi(x_1) \dots \phi(x_n) \rangle^{\mathcal{L}_{full}} \equiv \int \mathcal{D}\phi \left( \phi(x_1) \dots \phi(x_n) \right) \exp i \int dx \mathcal{L}_{full}(\phi, \Phi) \quad (4)$$

$$G(q_1, \dots, q_n, m, M) \equiv \int \left[ \prod_{i=1}^n dx_i \right] \exp i \left[ \sum_{i=1}^n q_i x_i \right] \times \langle T\phi(x_1) \dots \phi(x_n) \rangle^{\mathcal{L}_{full}} \quad (5)$$

renormalized in a minimal scheme. Let us consider the expansion of (5) in the inverse powers of  $M$  (Large Mass Expansion or LME). The leading order of the expansion can be written in the following form:

$$\langle T\phi(x_1) \dots \phi(x_n) \rangle^{\mathcal{L}_{full}(\phi, \Phi)} \underset{M \rightarrow \infty}{\simeq} \langle T\phi(x_1) \dots \phi(x_n) \rangle^{\mathcal{L}_{eff}(\phi)} + \mathcal{O}(M^{-1}) \quad (6)$$

or in momentum space

$$G(q, m, M) \underset{M \rightarrow \infty}{\simeq} \underline{G}(q, m, M) + \mathcal{O}(M^{-1}),$$

$$\underline{G}(q_1, \dots, q_n, m, M) \equiv \int \left[ \prod_{i=1}^n dx_i \right] \exp i \left[ \sum_{i=1}^n q_i x_i \right] \times \langle T\phi(x_1) \dots \phi(x_n) \rangle^{\mathcal{L}_{eff}}. \quad (7)$$

Here

$$\begin{aligned} \mathcal{L}_{eff} &= \mathcal{L}_{QCD}(\phi) + \delta\mathcal{L}_{QCD}(\phi) \\ \delta\mathcal{L}_{QCD} &= -\frac{1}{2}\delta\zeta_G (\partial_\mu G_\nu^a - \partial_\nu G_\mu^a) \partial_\mu G_\nu^a + \delta\zeta_{3G} (g_s f^{abc} (\partial_\mu G_\mu^a) G_\mu^b G_\mu^c) \\ &\quad -\frac{1}{4}\delta\zeta_{4G} (g_s^2 f^{abe} f^{cde} G_\mu^a G_\nu^b G_\mu^c G_\mu^d) + \delta\zeta_c (\partial_\mu \bar{c}^a \partial^\mu c^a) \\ &\quad + \delta\zeta_{q_L} (\bar{q}_L i \hat{\partial} q_L) + \delta\zeta_{q_R} (\bar{q}_R i \hat{\partial} q_R) - \delta\zeta_s (m \bar{q} q) \\ &\quad + \delta\zeta_{cGc} [g_s f^{abc} (\partial^\mu \bar{c}^a) c^b G_\mu^c] - \sum_{l=L,R} \delta\zeta_{q_l G q_l} (g_s \bar{q}_l T^a \hat{G}^a q_l) \end{aligned} \quad (8)$$

and coefficients  $\delta\zeta_i \equiv \zeta_i - 1$  in (8) are functions of  $M$  with logarithmic leading behaviour as  $M \rightarrow \infty$ . The form of asymptotic expansion (7) represents the perfect factorization property<sup>14</sup>, since heavy ( $M$ ) and light parameters ( $m$ ) are fully factorized. We consider only leading term in the expansion<sup>d</sup>, so all operators in  $\delta\mathcal{L}_{QCD}$  are renormalizable. Therefore, one can rescale the light fields

$$\underline{\phi} = \zeta_\phi^{1/2} \phi, \quad \zeta_\phi = 1 + \delta\zeta_\phi \quad (9)$$

and write the effective theory Lagrangian in terms of  $\underline{\phi}$

$$\mathcal{L}_{eff}(\phi, g_s, m, \xi) = \mathcal{L}_{QCD}(\underline{\phi}, \underline{g_s}, \underline{m}, \underline{\xi}), \quad (10)$$

where we also introduce new parameters which are related to the initial ones (2) by means of decoupling constants ( $l = L, R$ )

$$\underline{g_s} = \zeta_{g_s} g_s, \quad \underline{m} = \zeta_m m, \quad \underline{\xi} = \zeta_G \xi,$$

<sup>d</sup>One may increase the accuracy of the expansion in (6) by adding to  $\delta\mathcal{L}$  nonrenormalizable local operators built of  $\phi$  with the coefficients  $\mathcal{O}(M^{-a})$ ,  $a > 1$

where

$$\zeta_{g_s} = \zeta_{3G} \zeta_G^{-3/2} = \zeta_{4G}^{1/2} \zeta_G^{-1} = \zeta_{cGc} \zeta_c^{-1} \zeta_G^{-1/2} = \zeta_{q_l G q_l} \zeta_{q_l}^{-1} \zeta_G^{-1/2}, \quad (11a)$$

$$\zeta_m = \zeta_s \zeta_{q_L}^{-1/2} \zeta_{q_R}^{-1/2}. \quad (11b)$$

Due to the gauge invariance one should obtain the same result for  $\zeta_{g_s}$  in (11a) calculated from different vertices. Moreover, since dimensional regularization does not violate the gauge invariance, the longitudinal part of the gluon propagator is not renormalized. As a consequence, for the gauge-fixing parameter  $\xi$  one introduces the same decoupling constant as for the gluon field. According to (10), one can identify underlined parameters with those of QCD. Heavy degrees of freedom are said to be “decoupled”.

We should stress that  $g_s$ ,  $m$ , and  $\xi$  in the previous formulae are renormalized parameters and all the decoupling constants are finite. Evaluation of the constants for  $g_s$  and  $m$  requires a comparison of certain Green functions calculated with  $\mathcal{L}_{eff}$  with the lowest order expansion of the same functions calculated with  $\mathcal{L}_{full}$ . The matching is performed order by order in perturbation theory. Introducing

$$\zeta = 1 + \sum_{l=1}^{\infty} \delta\zeta^{(l)} \quad (12)$$

one can write the  $L$ -loop contribution to the decoupling constant  $\zeta_v$  for each vertex  $v$  from (8):

$$\delta\zeta_v^{(L)}(M) = \mathcal{P}_v \circ \left[ \text{As} \circ \Gamma_v^{(L)}(q, m, M) - \underline{\Gamma}_v^{(L)}(q, m, M) \right]. \quad (13)$$

Here  $\Gamma_v^{(L)}$  denotes the  $L$ -loop contribution to the renormalized one-particle-irreducible (1PI) Green function that corresponds to the vertex  $v$ . The operator  $\text{As}$  performs asymptotic expansion<sup>14</sup> of  $\Gamma_v^{(L)}$  calculated with  $\mathcal{L}_{full}$  up to the leading term in the inverse mass  $M$ . For calculation of  $\underline{\Gamma}_v^{(L)}$  one uses the  $(L-1)$ -loop effective theory Lagrangian that differs from (8) only by omission of all the terms in (12) with  $i \geq L$ . The appropriate projector  $\mathcal{P}_v$  applied to the Green function extracts the needed coefficient in front of the considered tensor (Lorentz, color, etc) structures (see examples below). All nonanalytical dependence on low mass scales is canceled in the left-hand side of (13) leading to

$$\delta\zeta_v^{(L)}(M) = \mathcal{P}_v \circ \mathcal{T} \circ \left[ \Gamma_v^{(L)}(q, m, M) - \underline{\Gamma}_v^{(L)}(q, m, M) \right], \quad (14)$$

where  $\mathcal{T}$  performs Taylor expansion in small mass  $m$  and external momenta  $q$ .

The procedure described above is straightforward, since it deals with the well-defined finite quantities but not the optimal one. Formulae (13) and (14) require evaluation of the Green functions within both the theories.

Let us recall that asymptotic expansion of a Feynman integral consists of the naive part, and the subgraph part. The naive part corresponds to Taylor expansion of the integrand in small parameters that cannot give rise to a nonanalytical dependence on low mass scales. Subgraphs restore missing terms in the result. The

calculation of asymptotic expansion can be rearranged in such a way (see below) that subgraphs of various diagrams contributing to the first term of (13) cancel the  $M$ -dependent contribution to the second term in the squared brackets. Taking into account that diagrams with all vertices coming from the QCD part of the Lagrangian contribute identically to both the terms of (13), decoupling constant calculations can be reduced to the evaluation of the naive part of LME of the diagrams with at least one heavy line.

There is another issue that has to be pointed out. Taylor expansion of the integrand may produce spurious IR divergences which can be avoided by a proper redefinition of dangerous terms in the sense of distributions<sup>15</sup>. In a dimensionally regularized form of the expansion the spurious divergences are canceled by the UV-divergent terms coming from the subgraphs. The rearrangement mentioned above is nothing else but addition of a necessary counter-term to the naive expansion and subtraction of the same expression from the subgraphs<sup>14</sup>.

A nice trick (see, e.g., Ref. 16) can be used to maintain the rearrangement automatically. One introduces decoupling constants for bare parameters  $A_0$

$$\underline{A}_0 = \zeta_{A,0} A_0 \quad (15)$$

and carries out matching at the bare level. In this case, naive Taylor expansion in small parameters is used to calculate the  $L$ -loop contribution to the bare decoupling constant

$$\delta\zeta_{v,0}^{(L)}(M) = \mathcal{P}_v \circ \mathcal{T} \circ \Gamma_{v,0}^{(L)}(q, m, M). \quad (16)$$

Then by the same formulae (11) one obtains relations (15) between bare parameters of the low- and high-energy theories. Definitely, this calculation introduces spurious IR divergences. However, they are completely canceled when one renormalizes the left- and right-hand sides of (15) in MS-scheme

$$\underline{A}_0 = \underline{Z}_A(\underline{A}) \underline{A}, \quad A_0 = Z_A(A, B) A, \quad (17)$$

$$\zeta_A = \left[ Z_A(A, B) \right] \left[ \underline{Z}_A(\underline{A}) \right]^{-1} \zeta_{A,0}(Z_A A, Z_B B, Z_M M). \quad (18)$$

Here we emphasized that renormalization constants  $Z_A$  and  $\underline{Z}_A$  are defined in different theories and depend on the parameters of the full  $(\underline{A}, \underline{B}, M)$  and effective theories  $(\underline{A})$ , respectively. Since  $\underline{A}$  enters into the right-hand side of (18), this is an equation that should be solved in perturbation theory.

Finally, we want to make a remark that a decoupling relation between MS-parameters can also be found<sup>17</sup> by considering momentum space ("physical") subtractions as an intermediate step. Indeed, the parameter  $A_{\text{MOM}}(Q)$  defined in MOM-scheme at some scale  $Q \ll M$  can be expressed either in terms of  $\underline{A}$  (effective theory) or in terms of  $A, B$  and  $M$  (full theory)

$$A_{\text{MOM}}(Q) = \underline{f}(\underline{A}, Q) = f(A, B, M, Q). \quad (19)$$

As it should be, it turns out that the relation between  $\underline{A}$  and the parameters of the full theory do not depend on  $Q$ .



### 3. Transition from $\overline{\text{DR}}$ to $\overline{\text{MS}}$ by decoupling of $\varepsilon$ -scalars

In dimensional regularization (DREG), the number of space-time dimensions is altered from four to  $d = 4 - 2\varepsilon$  which renders the loop integrations finite. It is clear, however, that if DREG is applied to a four-dimensional supersymmetric theory, the number of bosonic and fermionic degrees of freedom in supermultiplets is no longer equal, such that supersymmetry is explicitly broken. In order to avoid this problem, Dimensional Reduction has been suggested as an alternative regularization method<sup>1</sup>. Space-time is compactified to  $d = 4 - 2\varepsilon$  dimensions in DRED, such that the number of vector field components remains equal to four. Momentum integrations are  $d$ -dimensional, however, and divergences are parametrized in terms of  $1/\varepsilon$  poles, just like in DREG. Since it is assumed that  $\varepsilon > 0$ , the four-dimensional vector fields can be decomposed in terms of  $d$ -dimensional ones plus the so-called  $\varepsilon$ -scalars. The occurrence of these  $\varepsilon$ -scalars is, therefore, the only difference between DREG and DRED, so that all the calculational techniques developed for DREG are applicable also in DRED<sup>10</sup>.

Dimensional reduction of the four-dimensional  $\mathcal{L}_{QCD}$  leads to the following regularized form of QCD Lagrangian (see Appendix A)

$$\begin{aligned}\mathcal{L}_{QCD} &\rightarrow \mathcal{L}_{QCD}^{4-2\varepsilon} + \delta\mathcal{L}_{QCD}^\varepsilon \\ \delta\mathcal{L}_{QCD}^\varepsilon &= -\frac{1}{2} (D_\mu W^i)_a (D^\mu W_i)_a \\ &\quad - \frac{1}{4} g_s^2 f^{abc} f^{a\bar{b}\bar{c}} W_i^b W_j^c W_i^{\bar{b}} W_j^{\bar{c}} - g_s \bar{q} \gamma^i W_i^a T^a q,\end{aligned}\quad (20)$$

where  $W_i^a$  corresponds to  $\varepsilon$ -scalar fields and the indices  $i, j$  belong to the (space-like)  $2\varepsilon$ -subspace of the four-dimensional world. In nonsupersymmetric models there are several issues related to this Lagrangian. First of all, the last two terms in (20) are gauge-invariant separately<sup>18</sup>, and there is no symmetry that guarantees the same renormalization of the couplings that parametrize these vertices (see Appendix A). This leads to complications in the renormalization group analysis, since the running of the couplings (20) is different. Secondly, since  $W_i$  are scalars, all massive particles that couple to them contribute to the unphysical  $\varepsilon$ -scalar mass  $m_\varepsilon^2$ . In order to solve the first mentioned problem, one must introduce so-called evanescent<sup>18</sup> couplings for each vertex. To the second problem there are two approaches. One may either introduce  $\varepsilon$ -scalar mass explicitly in the Lagrangian (20) and renormalize it minimally ( $\overline{\text{DR}}$ -scheme) or use nonminimal counter-term to subtract radiative corrections to  $m_\varepsilon^2$  at each order of perturbation theory<sup>19</sup>. It turns out that these prescriptions give rise to the same final answer for the QCD observables. We have checked this explicitly by considering two-loop pole mass<sup>19,20</sup> of the quark in the  $\overline{\text{DR}}$  QCD.

In the context of SUSY QCD part of the MSSM the situation is different. All the dimensionless evanescent couplings are related to the gluon couplings by SUSY and, therefore, are not independent. This circumvents the first problem. However, two mentioned renormalization conditions for  $m_\varepsilon^2$  produce different answers. For

example, in  $\overline{\text{DR}}$  the pole mass of a scalar quark superpartner (squark) exhibits a quadratic dependence on  $m_\varepsilon^2$ . The authors of Ref. 21 proposed to redefine running squark masses  $m_q^2$  to absorb the unphysical contribution ( $\overline{\text{DR}}'$ -scheme). At the one-loop level<sup>e</sup> one has

$$(m_q^2)_{\overline{\text{DR}}'} = (m_q^2)_{\overline{\text{DR}}} - 2C_F \frac{\alpha_s}{4\pi} m_\varepsilon^2. \quad (21)$$

After such a redefinition one obtains the result which is independent of  $m_\varepsilon^2$ . The new scheme is equivalent to the prescription with a nonminimal counter-term<sup>19</sup>. This statement was also checked explicitly by considering heavy quark pole mass<sup>23</sup> in SUSY QCD as an observable.

Formula (21) and the reasoning that was used to obtain it allows one to interpret (21) as a first step towards decoupling of  $\varepsilon$ -scalars in the sense described in the previous section. Leading (but unphysical) ( $m_\varepsilon^2 \rightarrow \infty$ ) corrections to the pole mass of the squark are absorbed into redefinition of the corresponding mass parameter. One may go further and decouple  $\varepsilon$ -scalars completely. It seems useless in the context of SUSY QCD, since without  $\varepsilon$ -scalars one loses the advantages of DRED. However, it makes sense in the problem described in the paper. For nonsupersymmetric models  $\overline{\text{MS}}$ -scheme is natural in the sense that contrary to  $\overline{\text{DR}}$  it does not require introduction of evanescent couplings. Given the procedure (see Sec. 2), not only physical degrees of freedom can be decoupled but also unphysical  $\varepsilon$ -scalars. This leads to a direct relation between  $\overline{\text{MS}}$ -parameters of the effective theory and  $\overline{\text{DR}}$ -parameters of the full theory.

Using these simple arguments we calculate the relation between the  $b$ -quark running mass defined in the  $\overline{\text{MS}}$  QCD and its counter-part in  $\overline{\text{DR}}$  SUSY QCD. Before going to the final result, in the next section we want to demonstrate how known relations between  $\overline{\text{DR}}$  and  $\overline{\text{MS}}$  QCD parameters are reproduced.

#### 4. Toy example: matching $\overline{\text{MS}}$ QCD with $\overline{\text{DR}}$ QCD

Let us consider a model with  $(n_f - 1)$  massless quarks and only one massive quark with mass denoted by  $m$ . Thus, the task is to find two-loop relations of the following type:

$$g_s^{\overline{\text{MS}}} = g_s^{\overline{\text{DR}}} \times \zeta_{g_s}(\alpha_s^{\overline{\text{DR}}}, \alpha_y^{\overline{\text{DR}}}), \quad m^{\overline{\text{MS}}} = m^{\overline{\text{DR}}} \times \zeta_m(\alpha_s^{\overline{\text{DR}}}, \alpha_y^{\overline{\text{DR}}}) \quad (22)$$

where

$$\alpha_s = \frac{g_s^2}{4\pi}, \alpha_y = \frac{g_y^2}{4\pi}.$$

In (22) superscript tells us what kind of renormalization scheme is used and  $g_y$  is the evanescent coupling for  $\varepsilon$ -scalar interaction with quarks (see (A.18)). Usually,

<sup>e</sup>Two-loop result can be found in Ref. 22

(22) are solved in perturbation theory to obtain  $m^{\overline{\text{DR}}}$  and  $g_s^{\overline{\text{DR}}}$  as functions of  $\overline{\text{MS}}$ -parameters and evanescent ones. However, we use the form (22), since it is directly related to matching.

In this section, we consider “high-energy” theory with a Lagrangian  $\Delta\mathcal{L}$  (2)

$$\begin{aligned} \Delta\mathcal{L} = & -\frac{1}{2} (D_\mu W^i)_a (D^\mu W_i)_a + \frac{1}{2} m_\varepsilon^2 W_a^i W_i^a \\ & - \frac{1}{4} \sum_{r=1}^3 \lambda_r H_r^{abcd} W_i^a W_j^c W_i^b W_j^d - g_y \bar{q} \gamma^i W_i^a T^a q \end{aligned} \quad (23)$$

that describes “heavy” degrees of freedom. In (23) the mass of the unphysical scalars  $m_\varepsilon^2$  is explicitly introduced together with evanescent couplings  $g_y$  and  $\lambda_r$ . Tensors  $H_r^{abcd}$  have certain symmetric properties (see Appendix A) and define a color structure of the four-vertex.

First of all, to calculate bare decoupling constants that correspond to (8) we consider bare 1PI Green functions and their Taylor expansion in small masses and external momenta (16). In the  $\overline{\text{DR}}$  QCD left- and right-handed quarks are renormalized in the same way, so let us introduce  $\delta\zeta_q = \delta\zeta_{q_L} = \delta\zeta_{q_R}$  and  $\delta\zeta_{qGq} = \delta\zeta_{q_L G q_L} = \delta\zeta_{q_R G q_R}$ .

For calculation of  $\delta\zeta_{q,0}$  and  $\delta\zeta_{s,0}$  quark self-energy  $\Gamma_q$  is used

$$\begin{aligned} i\Gamma_q(\hat{p}, m, m_\varepsilon^2) &= \Sigma_v(p^2, m^2, m_\varepsilon^2) \hat{p} + \Sigma_s(p^2, m^2, m_\varepsilon^2) m \\ \delta\zeta_q &= \left( -i \frac{1}{4n_c p^2} \times \text{Tr} \hat{p} \Gamma_q \right) \Big|_{p, m=0} = -\Sigma_v(0, 0, m_\varepsilon^2) \end{aligned} \quad (24)$$

$$\delta\zeta_s = \left( +i \frac{1}{4n_c m} \times \text{Tr} \Gamma_q \right) \Big|_{p, m=0} = \Sigma_s(0, 0, m_\varepsilon^2), \quad (25)$$

where the trace is taken both over spinor and color indices and  $4n_c$  appears in the denominator because of chosen normalization ( $\text{Tr} \mathbf{1} = 4$  for Dirac algebra and  $\text{Tr} \mathbf{1} = n_c$  for color algebra).

Due to the gauge invariance not all the parameters from (8) are needed to find  $\delta\zeta_{g_s,0}$ . The simplest choice is to use the ghost-gluon vertex  $\Gamma_{cGc}$

$$g_s \delta\zeta_{cGc} = \left( \frac{1}{n_g} f^{abc} \times \frac{k_\mu}{k^2} \times \Gamma_{cGc}^{abc, \mu} \right) \Big|_{p, m=0}, \quad (26)$$

where  $a, b, c$  are gluon indices,  $f^{abc}$  corresponds to  $SU(3)$  structure constants,  $n_g = 8$  is the number of gluons,  $p$  denotes all external momenta and  $k_\mu$  is the momentum of incoming antighost (cf. (8)). One also needs to consider gluon and ghost self-energies

$$i\Gamma_G^{\mu\nu, ab}(p, m^2, m_\varepsilon^2) = -\delta^{ab} (g^{\mu\nu} p^2 - p^\mu p^\nu) \Pi_G(p^2, m^2, m_\varepsilon^2), \quad (27)$$

$$i\Gamma_c^{ab}(p^2, m^2, m_\varepsilon^2) = \delta^{ab} p^2 \Sigma_c(p^2, m^2, m_\varepsilon^2) \quad (28)$$

so

$$\delta\zeta_G = \left( +i \frac{\delta^{ab}}{n_g} \times \frac{1}{d-1} \left( g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \frac{1}{p^2} \times \Gamma_G^{\mu\nu, ab} \right) \Big|_{p,m=0} = -\Pi_G(0, 0, m_\varepsilon^2), \quad (29)$$

$$\delta\zeta_c = \left( -i \frac{\delta^{ab}}{n_g} \times \frac{1}{p^2} \times \Gamma_c^{ab} \right) \Big|_{p,m=0} = -\Sigma_c(0, 0, m_\varepsilon^2), \quad (30)$$

where  $d = 4 - 2\varepsilon$ , since we use  $d$ -dimensional metric tensor in (29). To check the final result for  $\delta\zeta_{g_s,0}$ , we also use the gluon-quark vertex  $\Gamma_{qGq}$

$$g_s \delta\zeta_{qGq} = \left( +i \frac{1}{4n_c C_F} \times \text{Tr } \gamma^\mu \Gamma_{qGq}^{\mu,a} T^a \right) \Big|_{p,m=0}, \quad (31)$$

where  $C_F = 4/3$  is a casimir of  $SU(3)$  and again the trace is taken over both spinor and color indices.

Direct evaluation of the diagrams that contribute to the one-loop decoupling constants gives

$$\delta\zeta_{qGq}^{(1)} = \alpha_y C_F \left( 1 + \varepsilon \left( \frac{1}{2} - L \right) \right), \quad (32)$$

$$\delta\zeta_q^{(1)} = \alpha_y C_F \left( 1 + \varepsilon \left( \frac{1}{2} - L \right) \right), \quad (33)$$

$$\delta\zeta_G^{(1)} = \frac{1}{3} \alpha_s C_A (1 - \varepsilon L) \quad (34)$$

$$\delta\zeta_s^{(1)} = 2\alpha_y C_F (1 + \varepsilon (1 - L)), \quad (35)$$

$$\delta\zeta_m^{(1)} = \delta\zeta_s^{(1)} - \delta\zeta_q^{(1)} = \alpha_y C_F \left( 1 + \varepsilon \left( \frac{3}{2} - L \right) \right), \quad (36)$$

$$\begin{aligned} \delta\zeta_{g_s}^{(1)} &= \delta\zeta_{qGq}^{(1)} - \delta\zeta_q^{(1)} - \frac{1}{2} \delta\zeta_G^{(1)} \\ &= \delta\zeta_{cGc}^{(1)} - \delta\zeta_c^{(1)} - \frac{1}{2} \delta\zeta_G^{(1)} = -\frac{1}{6} \alpha_s C_A (1 - \varepsilon L). \end{aligned} \quad (37)$$

Here all the coupling constants are considered to be defined in  $\overline{\text{DR}}$ ,  $C_A = 3$  is another casimir of  $SU(3)$ ,  $L \equiv \log m_\varepsilon^2/\mu^2$  and  $\delta\zeta_{cGc}^{(1)} = \delta\zeta_c^{(1)} = 0$ . Notice that at the one-loop level bare decoupling constants for the mass and the gauge coupling are finite as  $\varepsilon \rightarrow 0$  and exhibit a dependence on  $L$  only when  $\varepsilon \neq 0$ . As usual, since we want to consider two-loop matching we keep terms that are linear in  $\varepsilon$ .

Two-loop decoupling corrections look like

$$\begin{aligned}\delta\zeta_{qGq}^{(2)} &= \left(\frac{1}{\varepsilon} - 2L\right) \left(\alpha_y C_F \left[3\alpha_s C_F - \alpha_y (2C_F - C_A + n_f T_F)\right] + \frac{1}{8}\alpha_s^2 C_A^2\right) \\ &\quad + \alpha_y C_F \left(7\lambda_3 + 20\lambda_2 - \lambda_1 C_A + \alpha_s \left(\frac{3}{2}C_A + C_F\right)\right) \\ &\quad - \frac{1}{4}\alpha_s^2 C_A \left(\frac{11}{12}C_A + C_F\right) - \frac{1}{2}\alpha_y^2 C_F (C_F + n_f T_F)\end{aligned}\quad (38)$$

$$\delta\zeta_c^{(2)} = \frac{1}{4}\alpha_s^2 C_A^2 \left(-\frac{1}{2\varepsilon} + \frac{11}{24} + L\right) \quad (39)$$

$$\begin{aligned}\delta\zeta_q^{(2)} &= \left(\frac{1}{\varepsilon} - 2L\right) \alpha_y C_F \left[3\alpha_s C_F - \alpha_y (2C_F - C_A + n_f T_F)\right] \\ &\quad + \alpha_y C_F \left(7\lambda_3 + 20\lambda_2 - \lambda_1 C_A + \alpha_s \left(\frac{3}{2}C_A + C_F\right)\right) \\ &\quad - \frac{1}{2}\alpha_y^2 C_F (C_F + n_f T_F) - \frac{1}{4}\alpha_s^2 C_F C_A\end{aligned}\quad (40)$$

$$\begin{aligned}\delta\zeta_s^{(2)} &= \left(\frac{1}{\varepsilon} - 2L\right) C_F \left(2\alpha_y C_F \left[3\alpha_s C_F - \alpha_y (2C_F - C_A + n_f T_F)\right] + 6\alpha_s \alpha_y C_F\right) \\ &\quad + \frac{1}{2}\alpha_s^2 C_A \left) + \alpha_y C_F (14\lambda_3 + 40\lambda_2 - 2\lambda_1 C_A + 14\alpha_s C_F)\right. \\ &\quad \left.+ \alpha_y^2 C_F (3C_A - 6C_F - 2n_f T_F) - \frac{7}{6}\alpha_s^2 C_F C_A\right.\end{aligned}\quad (41)$$

$$\begin{aligned}\delta\zeta_G^{(2)} &= \alpha_s^2 C_A^2 \left(\frac{1}{4\varepsilon} + \frac{7}{8} - \frac{1}{2}L\right) + \alpha_s \alpha_y \left(\frac{2}{3}n_f T_F C_A - 2n_f T_F C_F\right) \\ &\quad + \frac{7}{3}\lambda_3 \alpha_s C_A + \frac{20}{3}\lambda_2 \alpha_s C_A - \frac{1}{3}\lambda_1 \alpha_s C_A^2\end{aligned}\quad (42)$$

and

$$\begin{aligned}\delta\zeta_m^{(2)} &= \delta\zeta_s^{(2)} - \delta\zeta_q^{(2)} - \delta\zeta_m^{(1)} \delta\zeta_q^{(1)} \\ &= \left(\frac{1}{\varepsilon} - 2L\right) C_F \left(\alpha_y C_F \left[3\alpha_s C_F - \alpha_y (2C_F - C_A + n_f T_F)\right] + \frac{1}{2}\alpha_s^2 C_A\right) \\ &\quad + \alpha_y C_F \left(7\lambda_3 + 20\lambda_2 - \lambda_1 C_A + \alpha_s \left(13C_F - \frac{3}{2}C_A\right)\right) \\ &\quad + \alpha_y^2 C_F \left(3C_A - \frac{13}{2}C_F - \frac{3}{2}n_f T_F\right) - \frac{11}{12}\alpha_s^2 C_F C_A,\end{aligned}\quad (43)$$

$$\begin{aligned}\delta\zeta_{g_s}^{(2)} &= \delta\zeta_{qGq}^{(2)} - \delta\zeta_q^{(2)} - \delta\zeta_q^{(1)} \delta\zeta_{g_s}^{(1)} - \frac{1}{2} \left(\delta\zeta_G^{(2)} + \delta\zeta_G^{(1)} \left[\delta\zeta_{g_s}^{(1)} + \delta\zeta_q^{(1)}\right] - \frac{1}{4} \left[\delta\zeta_G^{(1)}\right]^2\right) \\ &= \delta\zeta_{cGc}^{(2)} - \delta\zeta_c^{(2)} - \delta\zeta_c^{(1)} \delta\zeta_{g_s}^{(1)} - \frac{1}{2} \left(\delta\zeta_G^{(2)} + \delta\zeta_G^{(1)} \left[\delta\zeta_{g_s}^{(1)} + \delta\zeta_c^{(1)}\right] - \frac{1}{4} \left[\delta\zeta_G^{(1)}\right]^2\right) \\ &= \alpha_s \left(\frac{1}{6}\lambda_1 C_A^2 - \frac{7}{6}\lambda_3 C_A - \frac{10}{3}\lambda_2 C_A - \alpha_y n_f T_F \left(\frac{1}{3}C_A - C_F\right) - \frac{5}{8}\alpha_s C_A^2\right)\end{aligned}\quad (44)$$

In (43) and (44) we use perturbative expansion of (11). Clearly, (43) contains divergence and, therefore, the dependence on  $L$  arises at  $\mathcal{O}(\varepsilon^0)$ . Also there is a dependence on various  $\lambda_i$ . As it was noticed in Sec. 2 bare decoupling relations need to be properly renormalized (18). Let us demonstrate how this can be done at the two-loop level.

Recall again that  $A = \{g_s, m\}$  corresponds to high-energy theory parameters that have their counter-parts in the low-energy theory,  $B = \{g_y, \lambda_i\}$  denotes dimensionless parameters and  $M = m_\varepsilon$  represents large masses. Let us consider perturbative expansion of the bare decoupling and renormalization constants that enter into (18):

$$Z_A = 1 + \delta Z_A^{(1)}(A, B) + \delta Z_A^{(2)}(A, B), \quad (45)$$

$$\underline{Z}_A = 1 + \delta \underline{Z}_A^{(1)}(\underline{A}) + \delta \underline{Z}_A^{(2)}(\underline{A}), \quad (46)$$

$$\zeta_{A,0} = 1 + \delta \zeta_{A,0}^{(1)}(A_0, B_0, M_0) + \delta \zeta_{A,0}^{(2)}(A_0, B_0, M_0). \quad (47)$$

Substituting these quantities into (18) one obtains the following expression for renormalized decoupling constant  $\delta \zeta_A$ :

$$\delta \zeta_A^{(1)} = \delta Z_A^{(1)}(A, B) - \delta \underline{Z}_A^{(1)}(A) + \delta \zeta_{A,0}^{(1)}(A, B, M), \quad (48)$$

$$\begin{aligned} \delta \zeta_A^{(2)} = & \delta Z_A^{(2)}(A, B) - \delta \underline{Z}_A^{(2)}(A) - \delta \zeta_A^{(1)} \left( A \frac{\partial}{\partial A} \right) \delta \underline{Z}_A^{(1)}(A) \\ & + \left( \delta \underline{Z}_A^{(1)}(A) \right)^2 - \delta Z_A^{(1)}(A, B) \delta \underline{Z}_A^{(1)}(A) \\ & + \delta \zeta_{A,0}^{(1)}(A, B, M) \left( \delta Z_A^{(1)}(A, B) - \delta \underline{Z}_A^{(1)}(A) \right) \\ & + \delta \zeta_{A,0}^{(2)}(A, B, M) + \sum_{x=A,B,M} \left( \delta Z_x^{(1)} \left( x \frac{\partial}{\partial x} \right) \delta \zeta_{A,0}^{(1)}(A, B, M) \right). \end{aligned} \quad (49)$$

Consequently, to find the matching relations (22) we need to consider renormalization constants of the low-energy theory, i.e., QCD in  $\overline{\text{MS}}$ -scheme<sup>2,24</sup>,

$$\begin{aligned} Z_m^{\overline{\text{MS}}} = & 1 - 3C_F \frac{\alpha_s}{4\pi} \frac{1}{\varepsilon} + C_F \frac{\alpha_s^2}{(4\pi)^2} \left[ \frac{1}{\varepsilon^2} \left( \frac{11}{2}C_A + \frac{9}{2}C_F - 2n_f T_F \right) \right. \\ & \left. - \frac{1}{\varepsilon} \left( \frac{97}{12}C_A + \frac{3}{4}C_F - \frac{5}{3}n_f T_F \right) \right], \end{aligned} \quad (50)$$

$$\begin{aligned} Z_{g_s}^{\overline{\text{MS}}} = & 1 + \frac{\alpha_s}{4\pi} \frac{1}{\varepsilon} \left( \frac{2}{3}T_F n_f - \frac{11}{6}C_A \right) + \frac{\alpha_s^2}{(4\pi)^2} \left[ \frac{1}{\varepsilon^2} \left( \frac{121}{24}C_A^2 - \frac{11}{3}C_A T_F n_f \right. \right. \\ & \left. \left. + \frac{2}{3}n_f^2 T_F^2 \right) + \frac{1}{\varepsilon} \left( -\frac{17}{6}C_A^2 + \frac{5}{3}C_A T_F n_f + C_F T_F n_f \right) \right] \end{aligned} \quad (51)$$

together with renormalization constants of the full theory, i.e.,  $\overline{\text{DR}}$  QCD<sup>20</sup>,

$$\begin{aligned} Z_m^{\overline{\text{DR}}} &= 1 - 3C_F \frac{\alpha_s}{4\pi} \frac{1}{\varepsilon} - C_F \frac{\alpha_y}{4\pi} \left( -3C_F \frac{\alpha_s}{4\pi} \frac{1}{\varepsilon} + \frac{\alpha_y}{4\pi} \frac{1}{\varepsilon} (2C_F - C_A + T_F n_f) \right) \\ &\quad + C_F \frac{\alpha_s^2}{(4\pi)^2} \left[ \frac{1}{\varepsilon^2} \left( \frac{11}{2} C_A + \frac{9}{2} C_F - 2n_f T_F \right) \right. \\ &\quad \left. - \frac{1}{\varepsilon} \left( \frac{91}{12} C_A + \frac{3}{4} C_F - \frac{5}{3} n_f T_F \right) \right] \end{aligned} \quad (52)$$

$$Z_{g_y}^{\overline{\text{DR}}} = 1 - 3C_F \frac{\alpha_s}{4\pi} \frac{1}{\varepsilon} + \frac{\alpha_y}{4\pi} \frac{1}{\varepsilon} (2C_F - C_A + T_F n_f) \quad (53)$$

$$Z_{m_\varepsilon^2}^{\overline{\text{DR}}} = 1 - 3C_A \frac{\alpha_s}{4\pi} \frac{1}{\varepsilon} + 2n_f T_F \frac{\alpha_y}{4\pi} \frac{1}{\varepsilon} + (7\lambda_3 + 20\lambda_2 - \lambda_1 C_A) \frac{1}{\varepsilon}, \quad (54)$$

where we have omitted  $\overline{\text{DR}}$ -renormalization constant for the gauge coupling since it has the same form as (51). Notice that for our purpose we only need one-loop renormalization for the evanescent coupling  $\alpha_y$  and for the mass  $m_\varepsilon^2$ .

Given the knowledge of bare decoupling and renormalization constants in the effective and full theories, one can calculate renormalized decoupling corrections. Since one-loop renormalization for the gauge coupling and the quark mass coincides in  $\overline{\text{DR}}$  and  $\overline{\text{MS}}$ , the first two lines in (49) can be represented as

$$\begin{aligned} \frac{Z_m^{\overline{\text{DR}}}(\alpha_s, \alpha_y)}{Z_m^{\overline{\text{MS}}}(\alpha_s^{\overline{\text{MS}}})} &= 1 - C_F C_A \frac{\alpha_s^2}{(4\pi)^2} \frac{1}{2\varepsilon} + C_F C_A \frac{\alpha_s^2}{(4\pi)^2} L \\ &\quad - C_F \frac{\alpha_y}{4\pi} \left( -3C_F \frac{\alpha_s}{4\pi} \frac{1}{\varepsilon} + \frac{\alpha_y}{4\pi} \frac{1}{\varepsilon} (2C_F - C_A + T_F n_f) \right) \\ \frac{Z_g^{\overline{\text{DR}}}(\alpha_s, \alpha_y)}{Z_g^{\overline{\text{MS}}}(\alpha_s^{\overline{\text{MS}}})} &= 1 + \frac{C_A}{3} \frac{\alpha_s^2}{(4\pi)^2} \left( \frac{1}{\varepsilon} - L \right) \left( \frac{2}{3} T_F n_f - \frac{11}{6} C_A \right). \end{aligned}$$

The renormalization of the one-loop bare decoupling constants give rise to the following:

$$\begin{aligned} &2\delta Z_{g_y}^{(1)} \left( \alpha_y \frac{\partial}{\partial \alpha_y} \right) \delta \zeta_{m,0}^{(1)} + \delta Z_{m_\varepsilon^2}^{(1)} \left( m_\varepsilon^2 \frac{\partial}{\partial m_\varepsilon^2} \right) \delta \zeta_{m,0}^{(1)} \\ &= 2C_F \frac{\alpha_y}{4\pi} \left[ -3C_F \frac{\alpha_s}{4\pi} + \frac{\alpha_y}{4\pi} (2C_F - C_A + T_F n_f) \right] \left[ \frac{1}{\varepsilon} - L \right] \\ &\quad + C_F \frac{\alpha_y}{4\pi} \left[ \frac{\alpha_s}{4\pi} (3C_A - 9C_F) + \frac{\alpha_y}{4\pi} (6C_F - 3C_A + n_f T_F) \right. \\ &\quad \left. - 7\lambda_3 - 20\lambda_2 + \lambda_1 C_A \right], \end{aligned} \quad (55)$$

$$\begin{aligned} &2\delta Z_{g_s}^{(1)} \left( \alpha_s \frac{\partial}{\partial \alpha_s} \right) \delta \zeta_{g_s,0}^{(1)} + \delta Z_{m_\varepsilon^2}^{(1)} \left( m_\varepsilon^2 \frac{\partial}{\partial m_\varepsilon^2} \right) \delta \zeta_{g_s,0}^{(1)} \\ &= -\frac{1}{3} C_A \frac{\alpha_s^2}{(4\pi)^2} \left[ \frac{2}{3} T_F n_f - \frac{11}{6} C_A \right] \left[ \frac{1}{\varepsilon} - L \right] \\ &\quad + \frac{1}{6} C_A \frac{\alpha_s}{4\pi} \left( -3C_A \frac{\alpha_s}{4\pi} + 2n_f T_F \frac{\alpha_y}{4\pi} + 7\lambda_3 + 20\lambda_2 - \lambda_1 C_A \right). \end{aligned} \quad (56)$$

In the end of the day, one gets

$$m^{\overline{\text{MS}}} = m^{\overline{\text{DR}}} \left( 1 + \frac{\delta\zeta_m^{(1)}}{(4\pi)} + \frac{\delta\zeta_m^{(2)}}{(4\pi)^2} \right),$$

$$\delta\zeta_m^{(1)} = C_F \alpha_y, \quad (57)$$

$$\delta\zeta_m^{(2)} = -\frac{11}{12} C_F C_A \alpha_s^2 + C_F \alpha_s \alpha_y \left( 4C_F + \frac{3}{2} C_A \right) - \frac{1}{2} C_F \alpha_y^2 (C_F + n_f T_F), \quad (58)$$

$$g_s^{\overline{\text{MS}}} = g_s^{\overline{\text{DR}}} \left( 1 + \frac{\delta\zeta_{g_s}^{(1)}}{(4\pi)} + \frac{\delta\zeta_{g_s}^{(2)}}{(4\pi)^2} \right),$$

$$\delta\zeta_{g_s}^{(1)} = -\frac{1}{6} C_A \alpha_s, \quad (59)$$

$$\delta\zeta_{g_s}^{(2)} = -\frac{9}{8} C_A^2 \alpha_s^2 + n_f T_F C_F \alpha_s \alpha_y. \quad (60)$$

Obviously, the result is finite and coincides with the one that is known from literature (see, e.g., Ref. 25). All the dependence on  $L = \log m_\varepsilon^2/\mu^2$  and on evanescent couplings  $\lambda_i$  is canceled. Thus, by the explicit two-loop calculation we proved that the decoupling procedure well established in the context of perturbative QCD can be used not only for decoupling of heavy particles but also for  $\overline{\text{DR}} \rightarrow \overline{\text{MS}}$  transition.

## 5. The running mass of the b-quark: Matching QCD and SUSY QCD

In this section, we consider the SUSY QCD part of the MSSM as a full theory. The Lagrangian of SUSY QCD can be found, e.g., in Ref. 23. After dimensional reduction in addition to (20) there arise interactions of  $\varepsilon$ -scalars with squarks and gluinos (see Appendix B). The task again is to find a relation

$$m_b^{\overline{\text{MS}}}(\mu) = m_b^{\overline{\text{DR}}}(\mu) \times \zeta_{m_b} \left( \alpha_s^{\overline{\text{DR}}}, M^{\overline{\text{DR}}}, \mu \right), \quad (61)$$

where  $M^{\overline{\text{DR}}}$  corresponds to the  $\overline{\text{DR}}$ -renormalized masses of heavy particles. In the considered case

$$M = \{m_t, m_{\tilde{g}}, m_{\tilde{q}_i}\}, \quad q = \{u, d, c, s, t, b\}, \quad i = 1, 2, \quad (62)$$

where  $m_{\tilde{g}}$  denotes the gluino mass and  $m_{\tilde{q}_i}$  corresponds to squark masses.

Let us begin with the one-loop result for the mass and the gauge coupling (see,



e.g., Ref 26):

$$\delta\zeta_{g_s}^{(1)} = \frac{\alpha_s}{4\pi} \left( \frac{1}{3} \log \frac{m_t^2}{\mu^2} + \frac{C_A}{3} \log \frac{m_g^2}{\mu^2} + \frac{1}{12} \sum_q \sum_{i=1}^2 \log \frac{m_{\tilde{q}_i}^2}{\mu^2} - \frac{C_A}{6} \right), \quad (63)$$

$$\begin{aligned} \delta\zeta_{m_b}^{(1)} = & \frac{\alpha_s}{8\pi} C_F \left( 1 + \frac{m_g^2}{m_{\tilde{b}_1}^2 - m_g^2} + \frac{m_g^2}{m_{\tilde{b}_2}^2 - m_g^2} + 2 \log \frac{m_g^2}{\mu^2} \right. \\ & + \log \frac{m_{\tilde{b}_1}^2}{m_g^2} \left( 1 - \frac{m_g^4}{(m_{\tilde{b}_1}^2 - m_g^2)^2} - 2 \sin 2\theta_b \frac{m_g}{m_b} \frac{m_{\tilde{b}_1}^2}{m_{\tilde{b}_1}^2 - m_g^2} \right) \\ & \left. + \log \frac{m_{\tilde{b}_2}^2}{m_g^2} \left( 1 - \frac{m_g^4}{(m_{\tilde{b}_2}^2 - m_g^2)^2} + 2 \sin 2\theta_b \frac{m_g}{m_b} \frac{m_{\tilde{b}_2}^2}{m_{\tilde{b}_2}^2 - m_g^2} \right) \right). \end{aligned} \quad (64)$$

Here  $\theta_b$  is the bottom squark mixing angle. Unphysical  $\varepsilon$ -scalars contribute  $-\frac{\alpha_s}{4\pi} \frac{C_A}{6}$  to the gauge decoupling constant and  $\frac{\alpha_s}{4\pi} C_F$  to the quark mass decoupling constant. It should be noted that (64) is nothing else but the one-loop contribution to the pole mass of the quark from superparticles.

One may notice the dangerous dependence on  $m_b$  in (64). As it was stated in Sec. 2, decoupling constants should not depend on low mass scales. This contradiction is due to the fact that in the MSSM quarks acquire their masses after spontaneous breakdown of the electroweak symmetry (SSB). In spite of the fact that we neglect interactions parametrized by Yukawa couplings they obviously manifest themselves in quark masses. Due to the supersymmetry squark interactions with Higgs bosons are also parametrized by the same Yukawa couplings. After SSB squark quadratic Lagrangian receives a contribution proportional to the mass of a quark  $m_q$

$$\delta\mathcal{L}_{\tilde{q}\tilde{q}} = -m_q^2 (\tilde{q}_L^* \tilde{q}_L + \tilde{q}_R^* \tilde{q}_R) - m_q a_q (\tilde{q}_L^* \tilde{q}_R + \tilde{q}_R^* \tilde{q}_L), \quad a_q = A_q - \bar{\mu} \{\cot \beta, \tan \beta\}, \quad (65)$$

where  $\tilde{q}_{L,R}$  correspond to squark fields, and  $A_q, \bar{\mu}, \tan \beta$  are the MSSM parameters<sup>f</sup>. In the definition of  $a_q$  for up-squarks one has to choose  $\cot \beta$  and for down-squarks —  $\tan \beta$ . Usually one considers (65) as an additional contribution to the squark mass matrix and after diagonalization introduces mass eigenstates  $m_{\tilde{q}_1}^2, m_{\tilde{q}_2}^2$  and a mixing angle  $\theta_q$  that implicitly depend on  $m_q$ . If one takes into account that

$$\sin 2\theta_q = \frac{2m_q a_q}{m_{\tilde{q}_1}^2 - m_{\tilde{q}_2}^2} \quad (66)$$

it is possible to cancel dangerous powers of  $m_b$  in (64). However, it is not the end of the story, since the mass eigenvalues also depend on  $m_q$ .

There are two equivalent ways to obtain decoupling corrections that are formally independent of low mass scales. The first one is to reexpand (64) and (63) in  $m_b/M$ .

<sup>f</sup>We use  $\bar{\mu}$  to denote supersymmetric Higgs mixing parameter in order to distinguish it from the renormalization scale  $\mu$

The second one is to consider (65) as a part of the interaction Lagrangian from the very beginning. Clearly, insertion of the vertices from (65) in a Feynman diagram gives rise to a contribution that is proportional to some power of  $m_b$ . In the context of the asymptotic expansion only a finite number of these insertions has to be taken into account. For example, if one considers quark self-energy and the leading terms in the asymptotic expansion, it is sufficient to take into account only one insertion that mixes “left-handed” and “right-handed” squarks (see Fig. 2). This approach

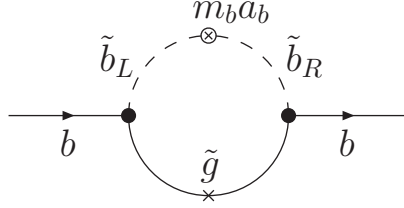


Fig. 2. Feynman diagram with one insertion of  $m_b a_b$  that contributes to the self-energy of the quark at the leading order in  $m_b/M$

allows one to keep all the dependence on  $m_b$  explicit and obtain decoupling constants that are independent of  $m_b$  and exhibit perfect factorization property. Nevertheless, the dependence on  $m_b$  of (63) and (64) is analytic as  $m_b \rightarrow 0$  and formally these expressions differ from the perfectly factorized ones only by the terms  $\mathcal{O}(m_b^2/M^2)$  that are negligible even for  $M \sim 0.1$  TeV. In our work we decided to keep the answer in the form that can be obtained from (64) by substitution of (66). This trick also works at the two-loop level.

The evaluation of the two-loop decoupling constant for the  $b$ -quark mass goes along the same lines as in the previous section. The important thing that has to be mentioned is that in SUSY QCD  $\delta\zeta_{qL} \neq \delta\zeta_{qR}$ . In this case<sup>g</sup>,

$$i\Gamma_q(\hat{p}, m) = \Sigma_L(p^2, m^2) \hat{p} P_L + \Sigma_R(p^2, m^2) \hat{p} P_R + \Sigma_s(p^2, m^2) m, \quad P_{L,R} = \frac{1 \mp \gamma_5}{2},$$

$$\delta\zeta_{qi} = \left( -i \frac{1}{2n_c p^2} \times \text{Tr } P_l \hat{p} \Gamma_q \right) \Big|_{p, m=0} = -\Sigma_l(0, 0), \quad l = L, R \quad (67)$$

<sup>g</sup>we suppress the dependence of the self-energy on large mass scales

and

$$\delta\zeta_m^{(1)} = \delta\zeta_s^{(1)} - \frac{1}{2} \left( \delta\zeta_{q_L}^{(1)} + \delta\zeta_{q_R}^{(1)} \right), \quad (68)$$

$$\begin{aligned} \delta\zeta_m^{(2)} = \delta\zeta_s^{(2)} - \frac{1}{2} \left( \delta\zeta_{q_L}^{(2)} + \delta\zeta_{q_R}^{(2)} + \delta\zeta_m^{(1)} \left[ \delta\zeta_{q_L}^{(1)} + \delta\zeta_{q_R}^{(1)} \right] \right. \\ \left. - \frac{1}{4} \left[ \delta\zeta_{q_L}^{(1)} - \delta\zeta_{q_R}^{(1)} \right]^2 \right). \end{aligned} \quad (69)$$

Since only quarks with the same chirality enter in the quark-gluon vertex, the expression for the gauge coupling decoupling constant (see (44)) is modified in a straightforward manner.

The calculation is performed in the  $\overline{\text{DR}}$ -scheme with an explicit mass term for the  $\varepsilon$ -scalars. Almost all needed renormalization constants for SUSY QCD can be found in Ref. 23. The only exception is the  $m_\varepsilon^2$  counter-term that looks like

$$Z_{m_\varepsilon^2} = 1 + \frac{\alpha_s}{4\pi} \frac{1}{\varepsilon} \left[ 2n_f T_F - 3C_A + \frac{1}{m_\varepsilon^2} \left( 2T_F \sum_{n_{\tilde{f}}} m_{\tilde{f}}^2 - 4T_f \sum_{n_f} m_f^2 - 2C_A m_{\tilde{g}}^2 \right) \right], \quad (70)$$

where  $n_{\tilde{f}}$  is used to denote the sum over different squarks and  $n_f$  corresponds to the summation over quark flavours. Notice that when SUSY is not broken by soft terms,  $m_{\tilde{g}}^2 = 0$  and  $m_q^2 = m_{q_i}^2$ , so the renormalization constant (70) and, thus, beta-function is homogeneous with respect to  $m_\varepsilon^2$  and one can safely put  $m_\varepsilon^2 = 0$  from the very beginning.

As it was mentioned in Sec. 2, it is possible to obtain the same expression by considering some observable that can be defined in both the effective and high-energy theories. For example, one can use the pole mass  $M_b$  as an intermediate quantity to find the relation between  $m_b^{\overline{\text{MS}}}$  and  $m_b^{\overline{\text{DR}}}$ . Since the two-loop SUSY QCD expression for  $M_b$  has been found earlier<sup>23</sup>, it is easy to calculate  $\delta\zeta_{m_b}^{(2)}$  given the knowledge of the one-loop decoupling constants (63)–(64). Let us briefly describe this approach, since we use it to cross-check our result.

Consider the two-loop relation between  $M_b$  and  $m_b^{\overline{\text{MS}}}$  calculated within the QCD<sup>2</sup>

$$M_b^{\overline{\text{MS}}} = m_b \left[ 1 + \sigma^{(1)}(m_b, \alpha_s) + \sigma^{(2)}(m_b, \alpha_s) \right], \quad m_b \equiv m_b^{\overline{\text{MS}}}, \alpha_s \equiv \alpha_s^{\overline{\text{MS}}}. \quad (71)$$

One can rewrite (71) in terms of SUSY QCD  $\overline{\text{DR}}$ -parameters by means of decoupling constants ( $m_b \equiv m_b^{\overline{\text{DR}}}$ ,  $g_s \equiv g_s^{\overline{\text{DR}}}$ )

$$\begin{aligned} M_b^{\overline{\text{MS}}} = m_b \left[ 1 + \left( \sigma^{(1)} + \delta\zeta_{m_b}^{(1)} \right) + \left( \sigma^{(2)} + \delta\zeta_{m_b}^{(2)} \right) \right. \\ \left. + \delta\zeta_{m_b}^{(1)} \left( 1 + m_b \frac{\partial}{\partial m_b} \right) \sigma^{(1)} + 2\delta\zeta_{g_s}^{(1)} \left( \alpha_s \frac{\partial}{\partial \alpha_s} \right) \sigma^{(1)} \right], \end{aligned} \quad (72)$$

where  $\sigma^{(1,2)}$  are the same functions of their arguments as in (71), i.e., they correspond to the diagrams with quarks and gluons only. As it was stated in Sec. 2, expression (72) allows one to approximate the result of the full theory.

If  $M_b^{\overline{\text{DR}}} = M_b^{\overline{\text{DR}}}(m_b^{\overline{\text{DR}}}, M^{\overline{\text{DR}}}, \dots)$  is the pole mass calculated within SUSY QCD, at the leading order of LME we have  $M_b^{\overline{\text{MS}}} = M_b^{\overline{\text{DR}}}$  and

$$\delta\zeta_{m_b}^{(2)} = \frac{M_b^{\overline{\text{DR}}} - m_b}{m_b} - \left( \sigma_1 + \delta\zeta_{m_b}^{(1)} \right) - \left( \sigma_2 + \delta\zeta_{m_b}^{(1)} \left( 1 + m_b \frac{\partial}{\partial m_b} \right) \sigma^{(1)} + 2\delta\zeta_{g_s}^{(1)} \left( \alpha_s \frac{\partial}{\partial \alpha_s} \right) \sigma^{(1)} \right). \quad (73)$$

Direct application of (73) gives rise to analytic expression for the  $m_b$  decoupling constant that is free from  $\log m_b/\mu$  but differs from the one obtained by the procedure described earlier. Careful investigation of the difference shows us that the discrepancy is due to the fact that both the results lack the perfect factorization property. The difference appears to be proportional to some power of  $\sin 2\theta_b$  and formally can be rewritten in such a way that it will be  $\mathcal{O}(m_b^2/M^2)$ . Indeed, numerical analysis shows that additional terms in the considered regions of the MSSM parameter space amount to  $10^{-3}$  % shift in the result.

The calculation of the corrections is carried out by means of a computer program written in FORM<sup>27</sup>. Two-loop bubble integrals that appear in LME are recursively reduced to a master-integral<sup>28</sup> by integration by the parts method<sup>29</sup>. The numerical evaluation of the master integral is carried out with the help of C++ library bubblesII<sup>30</sup>.

## 6. Results

Since we decouple all the heavy particles at the same time, this results in the huge expressions for the decoupling constants that depend on all the heavy mass scales of the model. Consequently, we will not present the answer in great detail as in Sec. 4 but just give a numerical impact of the result.

Evaluation of the corrections requires the knowledge of running MSSM parameters. However, the precise values are unknown, so one usually uses some hypothesis to reduce the parameter space of the model. The main uncertainty comes from the unknown soft terms. To reduce the number of free parameters, the so-called universality hypothesis is usually adopted, i.e., one assumes the universality or equality of various soft parameters at high energy scales. With the universality hypothesis one is left with the following set of free (mSUGRA<sup>31,32,33,34</sup>) parameters:

$$m_0, m_{1/2}, A_0 \text{ and } \tan\beta = \frac{v_2}{v_1}.$$

Here  $m_0$ ,  $m_{1/2}$  are universal scalar and fermion masses. They define mass splitting between the SM particles and their superpartners. Soft cubic interactions are parametrized by  $A_0$  and  $\tan\beta$  is the ratio of vacuum expectation values of the Higgs fields. Also the sign of  $\bar{\mu}$  is not fixed. In what follows we assume that  $\bar{\mu} > 0$ .

Usually, some computer code<sup>35,36,37,38</sup> is used to take an advantage of renormalization group method and calculate spectra and other observables. The universal boundary conditions are applied at some high energy scale  $M_{\text{GUT}}$ . However, it is

inconvenient to calculate low-energy observables in terms of parameters defined at  $M_{\text{GUT}}$ . One has to use the renormalization group to obtain the values of the corresponding parameters at the electroweak scale  $M_Z$  which is of our interest. There arises another complication, since for running one needs to know the values of dimensionless couplings at  $M_{\text{GUT}}$ . In contrast to soft terms gauge and Yukawa couplings are severely constrained by known electroweak physics, so natural boundary conditions for them are defined at  $M_Z$ . For most of the SM parameters these conditions are nothing else but relations of the type discussed in this work (1), so they are functions of (almost) *all* the parameters of the MSSM. To break this vicious circle, one usually makes a (reasonable) initial guess for unknown parameters either at  $M_{\text{GUT}}$  or at  $M_Z$  and after some iterations a stable solution for the equations is obtained.

In order to demonstrate our result, we present the values of two-loop corrections evaluated with running parameters given by the SOFTSUSY code<sup>35</sup>. The decoupling constant for  $m_b$  explicitly depends on the scale  $\mu$ . In order to reduce the uncertainty associated with large logarithms, one has to choose  $\mu \sim M$ . Indeed, Fig. 3 shows the dependence of two-loop corrections on the scale  $\mu$  for the specific point preferred by combined EGRET&WMAP constraints<sup>39</sup>. One sees that for  $\mu \sim 1$  TeV the calculated correction is about 1.5 %.

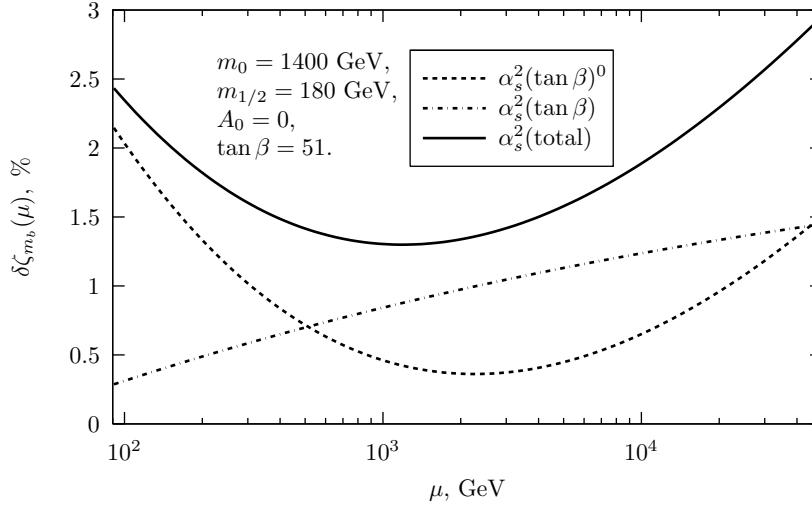


Fig. 3. The dependence of two-loop decoupling corrections  $\delta\zeta_{m_b}^{(2)}$  on the scale  $\mu$ . Lines marked by  $\alpha_s^2(\tan\beta)$  correspond to the contribution that is proportional to  $\tan\beta$ . Lines labeled by  $\alpha_s^2(\tan\beta)^0$  correspond to terms that lack such dependence on  $\tan\beta$ . Terms with  $\tan^n\beta$ ,  $n > 1$  turn out to be suppressed

Figure 3 also addresses another issue related to contributions that can be potentially enhanced by large  $\tan\beta$ . Since in our approach  $\tan\beta$  appears only through

mixing (66), it is easy to trace this dependence. Clearly, only first power of  $\tan\beta$  should be taken into account at the leading order of  $m_b/M$  expansion. From Fig. 3 one sees that even for large  $\tan\beta \simeq 51$  corrections  $\propto \tan\beta$  do not give a dominant contribution, so one should keep other terms in a careful analysis.

In the above-mentioned computer codes the relation between  $m_b^{\overline{\text{DR}}}(\mu)$  and  $m_b^{\overline{\text{MS}}}(\mu)$  is usually used at  $\mu = M_Z$ . In what follows we also employ this choice for matching. However, one should keep in mind that it is not the optimal scale for  $\delta\zeta_{m_b}^{(2)}$  evaluation.

The final aim of the calculation is to insert calculated correction to the  $m_b$  decoupling constant into the above-mentioned iterative process. We stress again that contrary to the  $t$ -quark case<sup>40</sup> the SUSY QCD contribution to the  $b$ -quark pole mass<sup>23</sup> should not be directly applied to the calculation of  $m_b^{\overline{\text{DR}}}$ . In Ref. 40, the two-loop SUSY QCD result for  $M_b$  was implicitly used as an estimate of the decoupling correction. At the one-loop level this is reasonable but it is not true at higher loops. Figure 4 shows a typical dependence of the corrections to  $\delta\zeta_{m_b}$  on  $m_{1/2}$  for certain values of other parameters of the model. For comparison we also plot pole mass corrections  $\delta z_{m_b} \equiv (M_b - m_b^{\overline{\text{DR}}})/m_b^{\overline{\text{DR}}}$ . It is clear that in the analysis of Ref. 23,  $\delta z_{m_b}$  overestimates  $\delta\zeta_{m_b}$ . Nevertheless, it was demonstrated<sup>23</sup> that for

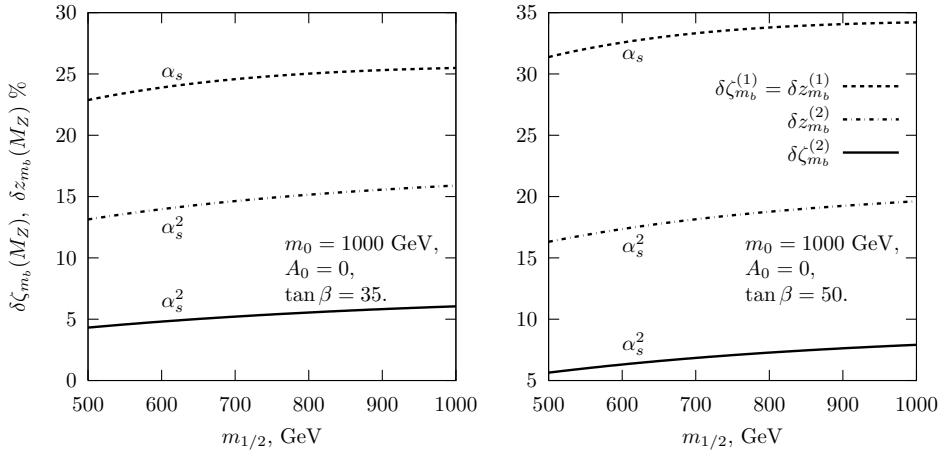


Fig. 4. Different SUSY QCD corrections to the  $b$ -quark pole mass  $M_b$  and the decoupling constant  $\zeta_{m_b}$  as functions of  $m_{1/2}$ . Here  $\delta z_{m_b} \equiv (M_b - m_b^{\overline{\text{DR}}})/m_b^{\overline{\text{DR}}}$  and  $\delta\zeta_{m_b} \equiv (m_b^{\overline{\text{MS}}} - m_b^{\overline{\text{DR}}})/m_b^{\overline{\text{DR}}}$ . At the leading order of Large Mass Expansion and at the one-loop level  $\delta\zeta_{m_b}^{(1)} = \delta z_{m_b}^{(1)}$ . However, for two-loop corrections  $\delta\zeta_{m_b}^{(2)} \neq \delta z_{m_b}^{(2)}$ .

a wide region of parameter space even overestimated SUSY QCD corrections do not influence superparticle spectrum significantly. They only become important for large values of  $\tan\beta$ , since in this case  $b$ -quark Yukawa coupling obtained from the running mass is also large. Indeed, Fig. 5 shows superparticle spectra for the EGRET&WMAP<sup>39</sup> point obtained by SOFTSUSY together with the shifts for the

masses after inclusion of our result in the code. One sees that for large  $\tan \beta$  two-loop corrections mostly influences a heavy Higgs spectrum<sup>41</sup>.

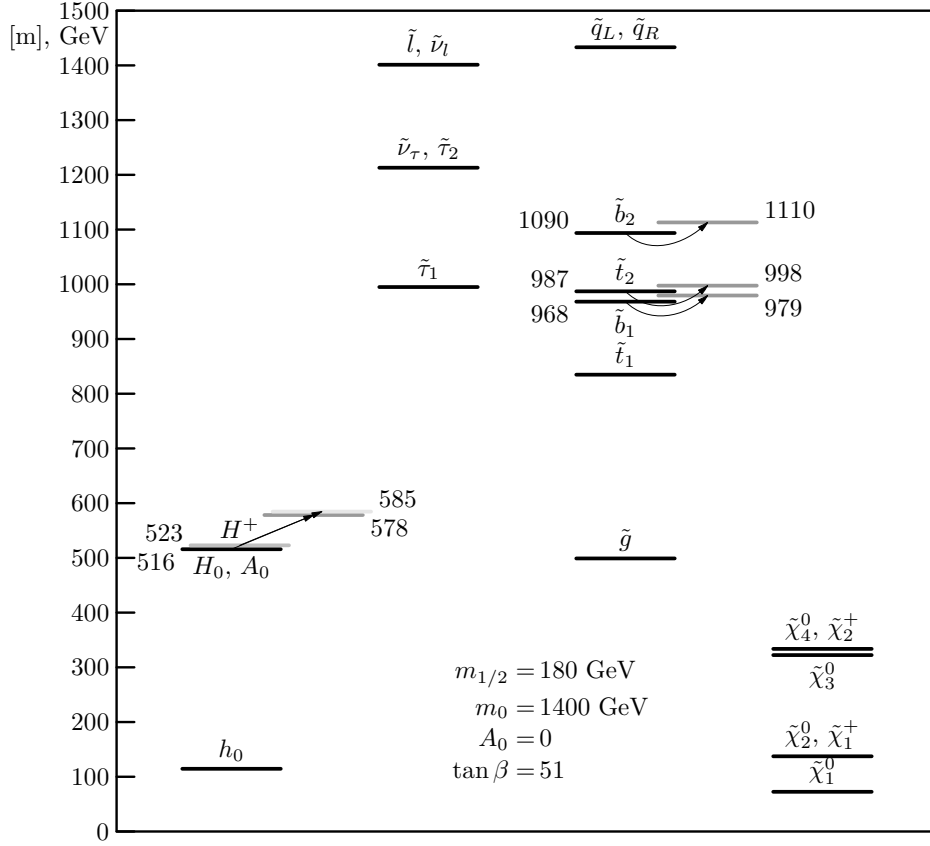


Fig. 5. Superparticle spectrum for the so-called EGRET&WMAP point of the MSSM parameter space. The shifts in mass values due to two-loop  $b$ -quark decoupling corrections are also presented (shifts less than one per cent are not shown).

## 7. Conclusions

The mass parameter of the  $b$ -quark plays an important role in phenomenological analysis of the MSSM. Strong interactions usually give rise to large radiative corrections to the quark mass and, thus, have to be calculated and taken into account. In this work we have proposed a method that allows one to find the value of the SUSY QCD  $\overline{\text{DR}}$ -running  $b$ -quark mass  $m_b^{\overline{\text{DR}}}$  directly from the corresponding value of  $\overline{\text{MS}}$ -mass  $m_b^{\overline{\text{MS}}}$  defined in the QCD. We consider the QCD as the low-energy effective theory of the more fundamental SUSY QCD and obtain the relation between  $m_b^{\overline{\text{MS}}}$  and  $m_b^{\overline{\text{DR}}}$  by decoupling of heavy particles.

The transition from  $\overline{\text{DR}}$  to  $\overline{\text{MS}}$  scheme can be achieved almost automatically by decoupling of unphysical  $\varepsilon$ -scalars together with physical squarks and gluinos. To justify the latter statement, decoupling of  $\varepsilon$ -scalars is considered in the context of  $\overline{\text{DR}}$  QCD and known relations between  $\overline{\text{DR}}$ - and  $\overline{\text{MS}}$ -parameters are obtained.

Applying a general matching procedure to the SUSY QCD case we calculate a two-loop contribution to the decoupling constant  $\zeta_{m_b}$  for the  $b$ -quark running mass. This in turn allows one to determine  $m_b^{\overline{\text{DR}}}$  more precisely from known SM input and implement three-loop running of the MSSM parameters (see Ref. 42) consistently. The numerical analysis of the correction and its impact on the spectrum is carried out. One, however, should keep in mind, that for the  $b$ -quark Yukawa interactions neglected in SUSY QCD give a sizable contribution to the pole mass<sup>43</sup>. Having in mind (73), one may try to calculate corrections to  $\zeta_{m_b}$  from the decoupling of Higgs bosons and their superpartners. We will study this issue elsewhere.

Finally, let us stress again the advantages and disadvantages of the proposed method. The main advantage seems obvious. One need not to consider evanescent couplings and their renormalization in nonsupersymmetric theories as, e.g., in Refs. 44,45. However, one has to pay some price for this simplification, since separate treatment of massive  $\varepsilon$ -scalars is required. For our problem we implemented the corresponding Feynman rules in FeynArts package and generated needed diagrams by computer.

Another obvious issue is a simultaneous decoupling of all heavy particles. This is reasonable only if the corresponding masses are of the same order, which may be not true for some SUSY scenarios, e.g. for Split SUSY<sup>46</sup>. In the latter case, a step-by-step decoupling is needed. Nevertheless, a  $\overline{\text{DR}} \rightarrow \overline{\text{MS}}$  transition is required at some stage and we think that this step can be carried out by decoupling of  $\varepsilon$ -scalars. It is reasonable to do this as soon as possible, since in this case no evanescent couplings appear in the effective nonsupersymmetric theory.

## 8. Acknowledgements

The author would like to thank D.I. Kazakov, A. Sheplyakov, and T. Hahn for fruitful discussions. Financial support from the Russian Foundation for Basic Research (grant # 05-02-17603) is kindly acknowledged.

## Appendix A. The $\varepsilon$ -scalars in the QCD

First of all, consider pure gauge QCD Lagrangian in four dimensions

$$\begin{aligned}\mathcal{L}_{QCD} &= -\frac{1}{4}F_{\mu\nu}^a F_a^{\mu\nu} \\ F_{\mu\nu}^a &= \partial_\mu G_\nu^a - \partial_\nu G_\mu^a - g_s f^{abc} G_\mu^b G_\nu^c\end{aligned}$$



Performing Dimensional Reduction from space-time dimension four to  $d = 4 - 2\varepsilon$  we should split four-vector into  $d$ -vector and so-called  $\varepsilon$ -scalars

$$\begin{aligned} d = 4 &\rightarrow d = 4 - 2\varepsilon \\ \mu &\rightarrow (\mu, \hat{\mu}) \\ g_{\mu\nu}^4 &\rightarrow (g_{\mu\nu}^{4-2\varepsilon}, g_{\hat{\mu}\hat{\nu}}^{2\varepsilon}) \\ G_\mu^a &\rightarrow (G_\mu^a, G_{\hat{\mu}}^a) \end{aligned}$$

“Coordinates” that correspond to  $2\varepsilon$  sub-space are assumed to be space-like, so  $g_{\hat{\mu}\hat{\mu}}^{2\varepsilon} = -1$  (no summation). In what follows we use Latin letters to denote  $2\varepsilon$  scalar indices and Greek letters for the Lorentz ones, i.e.,

$$g_{\hat{\mu}\hat{\nu}}^{2\varepsilon} \rightarrow g_{ij}, \quad g_{\mu\nu}^{4-2\varepsilon} \rightarrow g_{\mu\nu}, \quad G_{\hat{\mu}}^a \rightarrow W_i^a, \quad G_a^{\hat{\mu}} \rightarrow g^{ij}W_j^a = -W_i^a. \quad (\text{A.1})$$

Since all the fields after dimensional reduction do not depend on  $2\varepsilon$  coordinates, the corresponding derivatives (momenta) are zero. Consequently,

$$\begin{aligned} F_{\mu\nu}^a F_a^{\nu\nu} &\rightarrow F_{\mu\nu}^a F_a^{\mu\nu} + F_{\mu\hat{\nu}}^a F_a^{\mu\hat{\nu}} + F_{\hat{\mu}\nu}^a F_a^{\hat{\mu}\nu} + F_{\hat{\mu}\hat{\nu}}^a F_a^{\hat{\mu}\hat{\nu}} \\ F_{\mu\hat{\nu}}^a &= \partial_\mu G_{\hat{\nu}}^a - g_s f^{abc} G_\mu^b G_{\hat{\nu}}^c \\ F_{\hat{\mu}\nu}^a &= -\partial_\nu G_{\hat{\mu}}^a - g_s f^{abc} G_{\hat{\mu}}^b G_\nu^c = -F_{\nu\hat{\mu}}^a \\ F_{\hat{\mu}\hat{\nu}}^a &= -g_s f^{abc} G_{\hat{\mu}}^b G_{\hat{\nu}}^c \\ F_{\mu\hat{\nu}}^a F_a^{\mu\hat{\nu}} + F_{\hat{\mu}\nu}^a F_a^{\hat{\mu}\nu} &= 2F_{\mu\hat{\nu}}^a F_a^{\mu\hat{\nu}} \\ -\frac{1}{4}2F_{\mu\hat{\nu}}^a F_a^{\mu\hat{\nu}} &= -\frac{g^{ij}}{2}\partial_\mu W_i^a \partial^\mu W_j^a - \frac{g^{ij}}{2}g_s^2 f^{abc} f^{a\bar{b}\bar{c}} G_\mu^b W_i^c G_{\bar{b}}^\mu W_j^{\bar{c}} \\ &\quad + \frac{g^{ij}}{2}g_s f^{abc} G_b^\mu ((\partial_\mu W_i^a)W_j^c - (\partial_\mu W_i^c)W_j^a) \end{aligned}$$

and the Lagrangian of pure gauge QCD after dimensional reduction looks like

$$\begin{aligned} \mathcal{L}_{QCD}^{4-2\varepsilon} &= -\frac{1}{4}F_{\mu\nu}^a F_a^{\mu\nu} - \frac{g^{ij}}{2}\partial_\mu W_i^a \partial^\mu W_j^a + g^{ij}g_s f^{abc}\partial_\mu W_i^a G_b^\mu W_j^c \\ &\quad - \frac{g^{ij}}{2}g_s^2 f^{abc} f^{a\bar{b}\bar{c}} G_\mu^b W_i^c G_{\bar{b}}^\mu W_j^{\bar{c}} - \frac{g^{ij}g^{kl}}{4}g_s^2 f^{abc} f^{a\bar{b}\bar{c}} W_i^b W_k^c W_j^{\bar{b}} W_l^{\bar{c}} \\ &= -\frac{1}{4}F_{\mu\nu}^a F_a^{\mu\nu} - \frac{g^{ij}}{2}(\partial_\mu \delta^{ac} - g_s f^{abc} G_\mu^b) W_i^c (\partial^\mu \delta^{a\bar{c}} - g_s f^{a\bar{b}\bar{c}} G_{\bar{b}}^\mu) W_j^{\bar{c}} \\ &\quad - \frac{g^{ij}g^{kl}}{4}g_s^2 f^{abc} f^{a\bar{b}\bar{c}} W_i^b W_k^c W_j^{\bar{b}} W_l^{\bar{c}} \\ &= -\frac{1}{4}F_{\mu\nu}^a F_a^{\mu\nu} - \frac{g^{ij}}{2}(D_\mu W_i)_a (D^\mu W_j)_a - \frac{g^{ij}g^{kl}}{4}g_s^2 f^{abc} f^{a\bar{b}\bar{c}} W_i^b W_k^c W_j^{\bar{b}} W_l^{\bar{c}}, \end{aligned} \quad (\text{A.2})$$

where we introduced a covariant derivative

$$(D_\mu)_{ij} = \partial_\mu \delta_{ij} + ig_s T_{ij}^a G_\mu^a.$$

Here  $T_{ij}^a$  - generator of a gauge group in some representation. For the  $\varepsilon$ -scalars we have

$$D_\mu^{ac} = \partial_\mu \delta^{ac} + ig_s (-if^{bac}) G_\mu^b. \quad (\text{A.3})$$

Consider gauge transformations of the fields with infinitesimal parameter  $\omega^a$

$$\delta G_\mu^a = \partial_\mu \omega^a - g_s f^{abc} G_\mu^b \omega^c \quad (\text{A.4})$$

$$\delta W_i^a = -g_s f^{abc} W_i^b \omega^c \quad (\text{A.5})$$

All three terms in (A.2) are invariant under these transformations separately. As a consequence, couplings of gluon-gluon- $\varepsilon$ -scalar and gluon-gluon- $\varepsilon$ -scalar- $\varepsilon$ -scalar vertices are fixed by gauge invariance to be equal to  $g_s$ . On the contrary, gauge transformations do not mix  $\varepsilon$ -scalar four-vertex with something else.

The  $\varepsilon$ -scalar part of the action in momentum representation looks like

$$\begin{aligned} S_\varepsilon = & \int d^d p_1 \left[ \frac{p_1^2}{2} W_i^a(p_1) W_i^a(-p_1) \right] \\ & - i g^{ij} \frac{g_s}{2} f^{abc} \int d^d p_1 d^d p_2 (p_2^\mu - p_1^\mu) W_i^a(p_1) W_j^b(p_2) G_\mu^c(-p_1 - p_2) \\ & - g^{ij} \frac{g_s^2}{2} f^{abc} f^{ade} \int d^d p_1 d^d p_2 d^d p_3 W_i^c(p_1) W_j^e(p_2) G_\mu^b(p_3) G_d^\mu(-p_1 - p_2 - p_3) \\ & - g^{ij} g^{kl} \frac{g_s^2}{4} f^{abc} f^{ade} \int d^d p_1 d^d p_2 d^d p_3 W_i^b(p_1) W_k^c(p_2) W_j^d(p_3) W_l^e(-p_1 - p_2 - p_3). \end{aligned}$$

The corresponding Feynman rules can be derived from the action by taking a functional derivative with respect to the fields

$$i \frac{\delta^3 S}{\delta W_i^a(k_1) \delta W_j^b(k_2) \delta G_\mu^c(k_3)} = g_s f^{abc} \times g^{ij} (k_2^\mu - k_1^\mu), \quad (\text{A.6})$$

$$i \frac{\delta^4 S}{\delta W_i^a \delta W_j^b \delta G_\mu^c \delta G_\nu^d} = -i g_s^2 (f^{ace} f^{ebd} + f^{ade} f^{ebc}) \times g^{ij} g^{\mu\nu}, \quad (\text{A.7})$$

$$\begin{aligned} i \frac{\delta^4 S}{\delta W_i^a \delta W_j^b \delta W_k^c \delta W_l^d} = & -i g_s^2 (f^{ace} f^{ebd} + f^{ade} f^{ebc}) \times g^{ij} g^{kl} \\ & - i g_s^2 (f^{abe} f^{ecd} + f^{cbe} f^{ead}) \times g^{ik} g^{jl} \\ & - i g_s^2 (f^{abe} f^{edc} + f^{dbe} f^{eac}) \times g^{il} g^{jk}, \end{aligned} \quad (\text{A.8})$$

where an overall momentum conservation delta-function is implied.

In a general case, one may consider the following form of the  $\varepsilon$ -scalar four-vertex<sup>25</sup>:

$$\mathcal{L}_{\varepsilon\varepsilon\varepsilon\varepsilon} = -\frac{1}{4} \sum_{r=1}^R \lambda_r H_r^{abcd} W_i^a W_j^c W_i^b W_j^d \quad (\text{A.9})$$

Clearly, tensors  $H$  are symmetric under permutations  $a-b$ ,  $c-d$  and  $(a,b)-(c,d)$ . For the gauge group  $SU(N)$  the dimensionality  $R$  of the basis of rank-four tensors  $H_r^{abcd}$  that are symmetric with respect to  $(a,b)$  and  $(c,d)$  exchange is given by  $R=2$  for  $SU(2)$ ,  $R=3$  for  $SU(3)$  and  $R=4$  for  $SU(N)$ ,  $N \geq 4$ .

The Feynman rule for the vertex (A.9) with external  $\varepsilon$ -scalars reads

$$i \frac{\delta^4 S}{\delta W_i^a \delta W_j^b \delta W_k^c \delta W_l^d} = -i2 \sum_{r=1}^R \lambda_r (g^{ij} g^{kl} H_r^{abcd} + g^{ik} g^{jl} H_r^{acbd} + g^{il} g^{jk} H_r^{adbc}) \quad (\text{A.10})$$

One can choose  $H_r^{abcd}$  to be

$$H_1^{abcd} = \frac{1}{2} (f^{ace} f^{bde} + f^{ade} f^{bce}) \quad (\text{A.11a})$$

$$H_2^{abcd} = \delta^{ab} \delta^{cd} + (\delta^{ac} \delta^{bd} + \delta^{ad} \delta^{bc}) \quad (\text{A.11b})$$

$$H_3^{abcd} = \frac{1}{2} (\delta^{ac} \delta^{bd} + \delta^{ad} \delta^{bc}) - \delta^{ab} \delta^{cd} \quad (\text{A.11c})$$

If the QCD is embedded in a model with (softly broken) supersymmetry,  $\varepsilon$ -scalar four-vertex is related by (double) supersymmetry transformation to the corresponding gluon vertex. Consequently, if the symmetry is not explicitly broken by regularization and renormalization, couplings for gluon and  $\varepsilon$ -scalar four-vertices are renormalized in the same way, i.e.,  $\lambda_1 = g_s^2, \lambda_i = 0, i > 1$ .

We proceed with the fermion sector of the QCD. The interaction Lagrangian in four dimensions looks like

$$\delta \mathcal{L} = -g_s \bar{q} \gamma^\mu G_\mu^a T^a q, \quad (\text{A.12})$$

where  $T^a$  is a generator of  $SU(3)$  in fundamental representation. After dimensional reduction (A.12) induces interaction of the  $\varepsilon$ -scalars with quarks, i.e.,

$$\delta \mathcal{L}_\varepsilon = -g_s \bar{q} \gamma^i W_i^a T^a q \quad (\text{A.13})$$

It should be noticed that gamma matrices  $\gamma_i$  with index  $i$  from  $2\varepsilon$ -subspace, anti-commute with “ordinary” gamma-matrices that represent a vector with respect to  $d$ -dimensional Lorentz group. Another property is that the product of two identical gamma matrices  $\gamma_i$  is equal to  $-1$ . All these properties clearly come from the relations

$$\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu} \quad (\text{A.14})$$

$$\{\gamma_\mu, \gamma_i\} = 0 \quad (\text{A.15})$$

$$\{\gamma_i, \gamma_j\} = 2g_{ij} \quad (\text{A.16})$$

Again the term (A.13) alone is invariant under gauge transformations, so the renormalization of the corresponding coupling may not coincide with that of  $g_s$ . Consequently, we rewrite (A.13) in the following way:

$$\delta \mathcal{L}_\varepsilon = -g_y \bar{q} \gamma^i W_i^a T^a q, \quad (\text{A.17})$$

where  $g_y$  denotes evanescent Yukawa coupling<sup>18</sup>. It can be set to be equal to  $g_s$  at any scale. However, one should be careful trying to make a prediction at a different

scale due to different running of evanescent and real couplings. A Feynman rule for (A.17) reads

$$i \frac{\delta^3 S}{\delta_L \bar{q}_{\alpha_1}^{k_1} \delta_R q_{\alpha_2}^{k_2} \delta W_i^a} = -i g_y T_{k_1 k_2}^a \times \gamma_{\alpha_1 \alpha_2}^i, \quad (\text{A.18})$$

where  $\delta_{L(R)}$  denotes left (right) functional derivative,  $(k_1, k_2)$  are color indices and  $(\alpha_1, \alpha_2)$  correspond to Dirac spinor indices. In general, one should distinguish  $\gamma_i$  from  $\gamma^i = g^{ij} \gamma_j = -\gamma_i$ . However, in almost all practical calculations one “scalarize” the expression for a Feynman amplitude by contraction of its free indices with an appropriate projector. In a scalarized expression the relevant property is  $g^{ij} g_{ij} = 2\varepsilon$ .

Finally, there are gauge fixing and ghost terms in the Lagrangian of four-dimensional QCD

$$\delta \mathcal{L} = -\frac{1}{2\xi} (\partial_\mu G_a^\mu)^2 - \bar{c}^a \partial^\mu (\partial_\mu \delta^{ab} + g f^{abc} G_\mu^c) c^b. \quad (\text{A.19})$$

Clearly, after dimensional reduction these terms do not contribute to the interaction Lagrangian for the  $\varepsilon$ -scalars.

## Appendix B. The $\varepsilon$ -scalars in SUSY QCD

In SUSY QCD  $\varepsilon$ -scalars interact not only with quarks and gluons but also with squarks and gluinos. Actually, it is  $\varepsilon$ -scalars that balance the number of fermionic and bosonic degrees of freedom in the  $d$ -dimensional world.

As in the previous section, consider the four-dimensional form of the relevant part of the SUSY QCD Lagrangian

$$\delta \mathcal{L} = \frac{i}{2} \bar{\tilde{g}}^a D_\mu^{ab} \gamma^\mu \tilde{g}^b + \sum_{n=1,2} (D_\mu \tilde{q}_n)^* (D^\mu \tilde{q}_n). \quad (\text{B.1})$$

Here  $D_\mu^{ab}$  is a covariant derivative for gluinos  $\tilde{g}$  (see (A.3)) and  $D_\mu$  denotes a covariant derivative for squarks  $\tilde{q}_n$  that belong to fundamental representation of the color group. Notice that in (B.1) we do not write explicitly summation over quark flavours.

After dimensional reduction some of gluon fields that enter into covariant derivatives in (B.1) become  $\varepsilon$ -scalars. Therefore, gluino interaction with  $\varepsilon$ -scalars reads

$$\delta \mathcal{L}_\varepsilon = i \frac{g_s}{2} f^{abc} \bar{\tilde{g}}^a \gamma^i \tilde{g}^b W_i^c \quad (\text{B.2})$$

and the Feynman rule is

$$i \frac{\delta^3 S}{\delta_L \tilde{g}_{\alpha_1}^{a_1} \delta_R \tilde{g}_{\alpha_2}^{a_2} \delta W_i^{a_3}} = -g_s f^{a_1 a_2 a_3} \times \gamma_{\alpha_1 \alpha_2}^i. \quad (\text{B.3})$$

In (B.3) the factor  $1/2$  from (B.2) is canceled due to a majorana nature of gluino. For the squark- $\varepsilon$ -scalar interactions we have only four-vertices. Three-vertices inevitably

involve derivatives with respect to the coordinates that belong to  $2\varepsilon$  subspace and, therefore, vanish. Accordingly,

$$\delta\mathcal{L}_\varepsilon = g^{ij} g_s^2 \sum_{n=1,2} \tilde{q}_n^* T^a T^b \tilde{q}_n W_i^a W_j^b \quad (\text{B.4})$$

and the Feynman rule is

$$i \frac{\delta^4 S}{\delta (\tilde{q}^*)_{n_1}^{l_1} \delta \tilde{q}_{n_2}^{l_2} \delta W_{i_1}^{a_1} \delta W_{i_2}^{a_2}} = i g_s^2 (T^{a_1} T^{a_2} + T^{a_2} T^{a_1})_{l_1 l_2} \delta_{n_1 n_2} \times g^{ij}. \quad (\text{B.5})$$

Here again  $(l_1, l_2)$  are color indices and  $(n_1 n_2)$  numerate different squarks from (B.1). Generalization to the multi-flavour case is straightforward. Since strong interactions are flavour-blind, the “generalization” amounts to additional “flavour” Kronecker delta.

All needed Feynman rules are summarized in Table 1.

### Appendix C. FeynArts implementation of the $\varepsilon$ -scalar Lagrangian

The FeynArts package allows one to generate needed diagrams and Feynman amplitudes automatically. The MSSM has already been implemented in FeynArts (see Ref. 47). The model information is contained in two special files: The *generic model file* defines representation of the kinematical quantities. The *classes model file* sets up the particle content and specifies the actual couplings.

One of the crucial properties of  $\varepsilon$ -scalars is that they carry  $2\varepsilon$ -dimensional indices (one may say that we have  $2\varepsilon$  scalars). This property fixes “kinematical” structure of  $\varepsilon$ -scalar vertices, i.e., possible products of  $g^{ij}$  and other Lorentz objects which can appear in a vertex. Moreover, it does not depend on the group to which the  $\varepsilon$ -scalars belong. Consequently, the property can be realized at the *generic* level. For this purpose we write an addendum `LorentzEps.gen` for the generic model file `Lorentz.gen`.

The kinematical structure of vertices with  $\varepsilon$ -scalars is more like than of gauge bosons that of ordinary scalars. Instead of using a pre-defined generic scalar field `S`, it is convenient to introduce a new generic field `W` (`SE` in FeynArts). The field `W` represents *generic*  $\varepsilon$ -scalars and carries new kinematic index  $i = \text{Index}[\text{Escalar}]$ . For the field `Wi` we assume that there is no external wave function and a propagator has the form:

$$\langle W_i(-k) | W_j(k) \rangle = -i \frac{g^{ij}}{k^2 - m_\varepsilon^2}. \quad (\text{C.1})$$

The mass  $m_\varepsilon^2$  for the scalars is introduced due to the fact that there is no symmetry that keeps  $\varepsilon$ -scalars massless at each order of perturbation theory.

Let us summarize the generic kinematical structure of the couplings. We use the

same notation as in Ref. 48

$$C(W_i, W_j, W_k, W_l) = \vec{G}_{WWWW} \cdot \begin{pmatrix} g^{ij} g^{kl} \\ g^{ik} g^{jl} \\ g^{il} g^{jk} \end{pmatrix}_+ \quad (\text{C.2})$$

$$C(W_i, W_j, V_\mu, V_\nu) = G_{WWVV} \cdot (g^{ij} g^{\mu\nu})_+ \quad (\text{C.3})$$

$$C(W_i(k_1), W_j(k_2), V_\mu(k_3)) = G_{WWV} \cdot (g^{ij} (k_2^\mu - k_1^\mu))_- \quad (\text{C.4})$$

$$C(W_i, W_j, S, S) = G_{WWSS} \cdot (g^{ij})_+ \quad (\text{C.5})$$

$$C(F, F, W_i) = \vec{G}_{FFW} \cdot \begin{pmatrix} \gamma^i \omega_- \\ \gamma^i \omega_+ \end{pmatrix}_- \quad (\text{C.6})$$

$$C(W_i(k_1), W_j(k_2)) = \vec{G}_{WW} \cdot \begin{pmatrix} g^{ij} (k_1 k_2) \\ g^{ij} \end{pmatrix}_+ \quad (\text{C.7})$$

Here antisymmetric couplings are labeled by a subscript  $-$  and symmetric ones by a subscript  $+$ . The fields  $V_\mu, W_i, F, S$  correspond to generic vector,  $\varepsilon$ -scalar, fermion and ordinary scalar fields. Actual coupling vectors  $\vec{G}$  should be defined for each particular model.

Note that for the metric tensor  $g^{ij}$  and for the Dirac matrices  $\gamma^i$  no new symbols were defined. We use the following notation:

$$\begin{aligned} \gamma^i &= \text{DiracGamma}[\text{Index}[\text{Escalar}, i]], \\ g^{ij} &= \text{MetricTensor}[\text{Index}[\text{Escalar}, i], \text{Index}[\text{Escalar}, j]]. \end{aligned}$$

To implement gluon  $\varepsilon$ -scalars in the context of (SUSY) QCD a new classes model file is written `ESCALAR.mod`. Actually, the file only extends particle content and adds new couplings to the MSSM model `MSSMQCD.mod`. The generic (nonsupersymmetric) structure of vertices described in Appendix A is implemented. This allows one to use the same model file for the QCD and SUSY QCD. Actual coupling vectors for these models can be easily inferred from the expressions given above<sup>h</sup>.

## References

1. W. Siegel, “Supersymmetric dimensional regularization via dimensional reduction,” *Phys. Lett.* **B84** (1979) 193.
2. R. Tarrach, “The pole mass in perturbative qcd,” *Nucl. Phys.* **B183** (1981) 384.
3. A. S. Kronfeld, “The perturbative pole mass in QCD,” *Phys. Rev.* **D58** (1998) 051501, [hep-ph/9805215](#).
4. I. I. Y. Bigi, M. A. Shifman, N. G. Uraltsev, and A. I. Vainshtein, “The pole mass of the heavy quark. perturbation theory and beyond,” *Phys. Rev.* **D50** (1994) 2234–2246, [hep-ph/9402360](#).
5. M. Beneke and V. M. Braun, “Heavy quark effective theory beyond perturbation theory: Renormalons, the pole mass and the residual mass term,” *Nucl. Phys.* **B426** (1994) 301–343, [hep-ph/9402364](#).

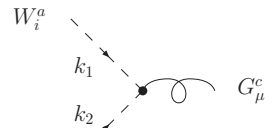
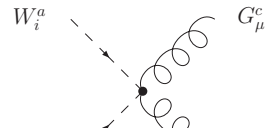
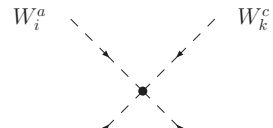
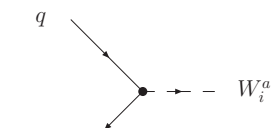
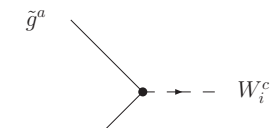
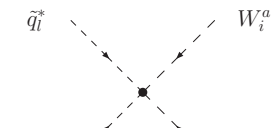
<sup>h</sup> See <http://theor.jinr.ru/~bednya/pmwiki/pmwiki.php?n=Main.Escalars>

6. H. Baer, J. Ferrandis, K. Melnikov, and X. Tata, “Relating bottom quark mass in  $\overline{\text{dr}}$ -bar and  $\overline{\text{ms}}$ -bar regularization schemes,” *Phys. Rev.* **D66** (2002) 074007, [hep-ph/0207126](#).
7. **Particle Data Group** Collaboration, W. M. Yao *et al.*, “Review of particle physics,” *J. Phys.* **G33** (2006) 1–1232.
8. I. Z. Rothstein, “Tasi lectures on effective field theories,” [hep-ph/0308266](#).
9. H. Georgi, “Effective field theory,” *Ann. Rev. Nucl. Part. Sci.* **43** (1993) 209–252.
10. R. V. Harlander, D. R. T. Jones, P. Kant, L. Mihaila, and M. Steinhauser, “Four-loop beta function and mass anomalous dimension in dimensional reduction,” *JHEP* **12** (2006) 024, [hep-ph/0610206](#).
11. H. Baer, J. Ferrandis, S. Kraml, and W. Porod, “On the treatment of threshold effects in susy spectrum computations,” *Phys. Rev.* **D73** (2006) 015010, [hep-ph/0511123](#).
12. T. Hahn, “Generating feynman diagrams and amplitudes with feynarts 3,” *Comput. Phys. Commun.* **140** (2001) 418–431, [hep-ph/0012260](#).
13. T. Appelquist and J. Carazzone, “Infrared singularities and massive fields,” *Phys. Rev.* **D11** (1975) 2856.
14. F. V. Tkachov, “Euclidean asymptotic expansions of green functions of quantum fields. 1. expansions of products of singular functions,” *Int. J. Mod. Phys.* **A8** (1993) 2047–2117, [hep-ph/9612284](#).
15. F. V. Tkachov, “Theory of asymptotic operation. a summary of basic principles,” *Sov. J. Part. Nucl.* **25** (1994) 649, [hep-ph/9701272](#).
16. K. G. Chetyrkin, B. A. Kniehl, and M. Steinhauser, “Decoupling relations to  $\mathcal{O}(\alpha_s^3)$  and their connection to low-energy theorems,” *Nucl. Phys.* **B510** (1998) 61–87, [hep-ph/9708255](#).
17. W. Bernreuther and W. Wetzel, “Decoupling of heavy quarks in the minimal subtraction scheme,” *Nucl. Phys.* **B197** (1982) 228.
18. I. Jack, D. R. T. Jones, and K. L. Roberts, “Dimensional reduction in nonsupersymmetric theories,” *Z. Phys.* **C62** (1994) 161–166, [hep-ph/9310301](#).
19. L. V. Avdeev and M. Y. Kalmykov, “Pole masses of quarks in dimensional reduction,” *Nucl. Phys.* **B502** (1997) 419–435, [hep-ph/9701308](#).
20. P. Marquard, L. Mihaila, J. H. Piclum, and M. Steinhauser, “Relation between the pole and the minimally subtracted mass in dimensional regularization and dimensional reduction to three-loop order,” [hep-ph/0702185](#).
21. I. Jack, D. R. T. Jones, S. P. Martin, M. T. Vaughn, and Y. Yamada, “Decoupling of the epsilon scalar mass in softly broken supersymmetry,” *Phys. Rev.* **D50** (1994) 5481–5483, [hep-ph/9407291](#).
22. S. P. Martin, “Two-loop effective potential for a general renormalizable theory and softly broken supersymmetry,” *Phys. Rev.* **D65** (2002) 116003, [hep-ph/0111209](#).
23. A. Bednyakov, A. Onishchenko, V. Velizhanin, and O. Veretin, “Two-loop  $\mathcal{O}(\alpha_s^2)$  mssm corrections to the pole masses of heavy quarks,” *Eur. Phys. J.* **C29** (2003) 87–101, [hep-ph/0210258](#).
24. J. Fleischer, F. Jegerlehner, O. V. Tarasov, and O. L. Veretin, “Two-loop QCD corrections of the massive fermion propagator,” *Nucl. Phys.* **B539** (1999) 671–690, [hep-ph/9803493](#).
25. R. Harlander, P. Kant, L. Mihaila, and M. Steinhauser, “Dimensional reduction applied to qcd at three loops,” *JHEP* **09** (2006) 053, [hep-ph/0607240](#).
26. D. M. Pierce, J. A. Bagger, K. T. Matchev, and R.-j. Zhang, “Precision corrections in the minimal supersymmetric standard model,” *Nucl. Phys.* **B491** (1997) 3–67, [hep-ph/9606211](#).

27. J. A. M. Vermaseren, “New features of form,” `math-ph/0010025`.
28. A. I. Davydychev and J. B. Tausk, “Two loop selfenergy diagrams with different masses and the momentum expansion,” *Nucl. Phys.* **B397** (1993) 123–142.
29. K. G. Chetyrkin and F. V. Tkachov, “Integration by parts: The algorithm to calculate beta functions in 4 loops,” *Nucl. Phys.* **B192** (1981) 159–204.
30. A. Sheplyakov, “bubblesii, a c++ library for analytical and numerical evaluation of 2-loop vacuum integrals.” The source code can be obtained from <http://theor.jinr.ru/~varg/dist>.
31. A. H. Chamseddine, R. Arnowitt, and P. Nath, “Locally supersymmetric grand unification,” *Phys. Rev. Lett.* **49** (1982) 970.
32. R. Barbieri, S. Ferrara, and C. A. Savoy, “Gauge models with spontaneously broken local supersymmetry,” *Phys. Lett.* **B119** (1982) 343.
33. L. J. Hall, J. D. Lykken, and S. Weinberg, “Supergravity as the messenger of supersymmetry breaking,” *Phys. Rev.* **D27** (1983) 2359–2378.
34. P. Nath, R. Arnowitt, and A. H. Chamseddine, “Gauge hierarchy in supergravity guts,” *Nucl. Phys.* **B227** (1983) 121.
35. B. C. Allanach, “Softsusy: A c++ program for calculating supersymmetric spectra,” *Comput. Phys. Commun.* **143** (2002) 305–331, `hep-ph/0104145`.
36. W. Porod, “Spheno, a program for calculating supersymmetric spectra, susy particle decays and susy particle production at e+ e- colliders,” *Comput. Phys. Commun.* **153** (2003) 275–315, `hep-ph/0301101`.
37. A. Djouadi, J.-L. Kneur, and G. Moultaka, “Suspect: A fortran code for the supersymmetric and higgs particle spectrum in the mssm,” *Comput. Phys. Commun.* **176** (2007) 426–455, `hep-ph/0211331`.
38. F. E. Paige, S. D. Protopopescu, H. Baer, and X. Tata, “Isajet 7.69: A monte carlo event generator for p p, anti-p p, and e+ e- reactions,” `hep-ph/0312045`.
39. W. de Boer, C. Sander, V. Zhukov, A. V. Gladyshev, and D. I. Kazakov, “The supersymmetric interpretation of the egret excess of diffuse galactic gamma rays,” *Phys. Lett.* **B636** (2006) 13–19, `hep-ph/0511154`.
40. A. Bednyakov, D. I. Kazakov, and A. Sheplyakov, “On the two-loop  $\mathcal{O}(\alpha_s^2)$  corrections to the pole mass of the t-quark in the mssm,” *Phys. Atom. Nucl.* **70** (2007) 198–203, `hep-ph/0507139`.
41. B. C. Allanach, S. Kraml, and W. Porod, “Theoretical uncertainties in sparticle mass predictions from computational tools,” *JHEP* **03** (2003) 016, `hep-ph/0302102`.
42. I. Jack, D. R. T. Jones, and A. F. Kord, “Snowmass benchmark points and three-loop running,” *Ann. Phys.* **316** (2005) 213–233, `hep-ph/0408128`.
43. A. Bednyakov and A. Sheplyakov, “Two-loop  $\mathcal{O}(\alpha_s y^2)$  and  $\mathcal{O}(y^4)$  mssm corrections to the pole mass of the b-quark,” *Phys. Lett.* **B604** (2004) 91–97, `hep-ph/0410128`.
44. R. Harlander, L. Mihaila, and M. Steinhauser, “Two-loop matching coefficients for the strong coupling in the mssm,” *Phys. Rev.* **D72** (2005) 095009, `hep-ph/0509048`.
45. R. V. Harlander, L. Mihaila, and M. Steinhauser, “Running of  $\alpha_s$  and  $m_b$  in the mssm,” `arXiv:0706.2953 [hep-ph]`.
46. G. F. Giudice and A. Romanino, “Split supersymmetry,” *Nucl. Phys.* **B699** (2004) 65–89, `hep-ph/0406088`.
47. T. Hahn and C. Schappacher, “The implementation of the minimal supersymmetric standard model in feynarts and formcalc,” *Comput. Phys. Commun.* **143** (2002) 54–68, `hep-ph/0105349`.
48. T. Hahn, *FeynArts 3.2. User’s guide*.



Table 1. Feynman rules for gluon  $\varepsilon$ -scalars. All momenta are incoming. Metric tensor  $g^{ij}$  corresponds to  $2\varepsilon$ -dimensional space,  $g^{ii} = -1$  (no summation). Gamma matrices  $\gamma^i$  carry  $2\varepsilon$ -dimensional indices. Generators and structure constants of  $SU(3)$  are denoted by  $T^a$  and  $f^{abc}$ , respectively. For the definition of  $H_r^{abcd}$  see (A.11).

$W_i^a \bullet \text{---} \text{---} \text{---} \bullet W_j^b$	$-ig^{ij} \delta^{ab} (k^2 - m_\varepsilon^2)^{-1}$
	$g_s f^{abc} \times g^{ij} (k_2^\mu - k_1^\mu)$
	$-ig_s^2 (f^{ace} f^{ebd} + f^{ade} f^{ebc}) \times g^{\mu\nu} g^{ij}$
	$-i2 \sum_{r=1}^3 \lambda_r (g^{ij} g^{kl} H_r^{abcd} + g^{ik} g^{jl} H_r^{acbd} + g^{il} g^{jk} H_r^{adbc})$
	$-ig_y T^a \times \gamma^i$
	$-g_s f^{abc} \times \gamma^i$
	$g_s^2 (T^a T^b + T^b T^a) \delta_{ln} \times g^{ij}$